Ορθή κυλινδρική ισαπέχουσα προβολή (1/4)

Σφαίρα

$$x = R\,\lambda \qquad \lambda = \frac{x}{R},$$

$$y = R\,\varphi \qquad \varphi = \frac{y}{R}.$$

$$x = a\,\lambda \qquad y = \boldsymbol{M}$$

$$\boldsymbol{M} = \mathrm{a}\,(1-e^2)\,[\boldsymbol{M}_0\varphi - \boldsymbol{M}_2\sin 2\varphi + \boldsymbol{M}_4\sin 4\varphi - \boldsymbol{M}_6\sin 6\varphi]$$

Ελλειψοειδές

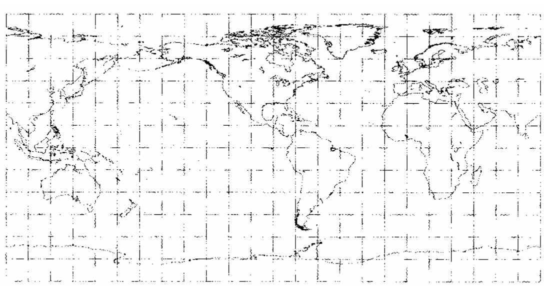
$$\boldsymbol{M}_0 = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + \frac{175}{256}e^6 + \dots$$

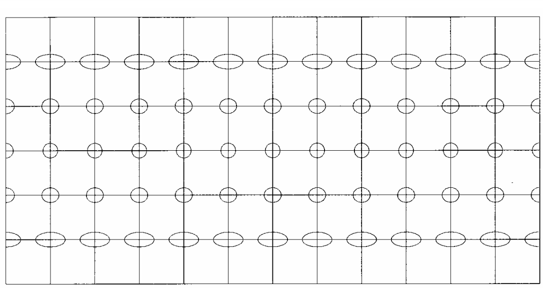
$$\boldsymbol{M}_2 = \frac{3}{8}e^2 + \frac{15}{32}e^4 + \frac{525}{1024}e^6 + \dots$$

$$\boldsymbol{M}_4 = \frac{15}{256}e^4 + \frac{105}{1024}e^6 + \dots$$

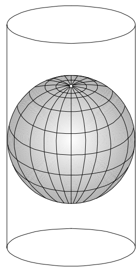
$$\boldsymbol{M}_6 = \frac{35}{3072}e^6 + \dots$$

Ορθή κυλινδρική ισαπέχουσα προβολή (2/4)





Ορθή κυλινδρική τετραγωνική προβολή 3/4



$$x = R\,\lambda$$

$$y = R\,\varphi$$

$$m_m = \frac{\mathrm{d}s_m}{\mathrm{d}m} = \frac{\mathrm{d}y}{R\,\mathrm{d}\varphi} = \frac{R\,\mathrm{d}\varphi}{R\,\mathrm{d}\varphi} = 1$$

$$m_p = \frac{\mathrm{d}s_p}{\mathrm{d}p} = \frac{\mathrm{d}x}{R\cos\varphi\,\mathrm{d}\lambda} = \frac{R\,\mathrm{d}\lambda}{R\cos\varphi\,\mathrm{d}\lambda} = \frac{1}{\cos\varphi}$$

$$M = \frac{1}{\cos\varphi} \qquad \sin E = \tan^2\frac{\varphi}{2}$$

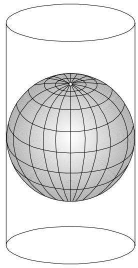
Ορθή κυλινδρική ισαπέχουσα προβολή (4/4)

$$m_\mathrm{m} = \frac{ds_\mathrm{m}}{dm} = \frac{dy}{d\boldsymbol{M}} = \frac{d\boldsymbol{M}}{d\boldsymbol{M}} = 1,$$

$$m_\mathrm{p} = \frac{ds_\mathrm{p}}{dp} = \frac{dx}{N\cos\varphi\,d\lambda} = \frac{\mathrm{a}\,d\lambda}{\dfrac{\mathrm{a}}{\sqrt{1-e^2\sin^2\varphi}}\cos\varphi\,d\lambda} = \frac{\sqrt{1-e^2\sin^2\varphi}}{\cos\varphi}.$$

$$M = \frac{1}{\cos\varphi}\ \text{και}\ \sin E = \tan^2\frac{\varphi}{2}$$

Ορθή Μερκατορική προβολή *1/3*



$$m_m = m_p \Rightarrow \frac{ds_m}{dm} = \frac{ds_p}{dp} \Rightarrow \frac{dy}{R\,d\varphi} = \frac{dx}{R\cos\varphi\,d\lambda}$$

$$dx = R\,d\lambda$$

$$dy = \frac{R}{\cos\varphi}d\varphi \Rightarrow y = R\int_0^{\varphi}\frac{1}{\cos\varphi}d\varphi \Rightarrow$$

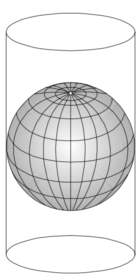
$x = R\,\lambda$

$y = ?$

$$y = R\ln\tan\left(45^{\circ} + \frac{\varphi}{2}\right)$$

$$m_m = m_p = \frac{1}{\cos\varphi} \qquad M = \frac{1}{\cos^2\varphi}$$

Ορθή Μερκατορική προβολή *2/3*



$$m_m = m_p \Rightarrow \frac{ds_m}{dm} = \frac{ds_p}{dp} \Rightarrow \frac{dy}{\rho\,d\varphi} = \frac{dx}{N\cos\varphi\,d\lambda}$$

$$dy = \frac{\rho\,a}{N\cos\varphi}d\varphi = \frac{a\,(1-e^2)}{(1-e^2\sin^2\varphi)\cos\varphi}d\varphi$$

$$\Rightarrow y = a\int_0^{\varphi}\frac{1-e^2}{(1-e^2\sin^2\varphi)\cos\varphi}d\varphi$$

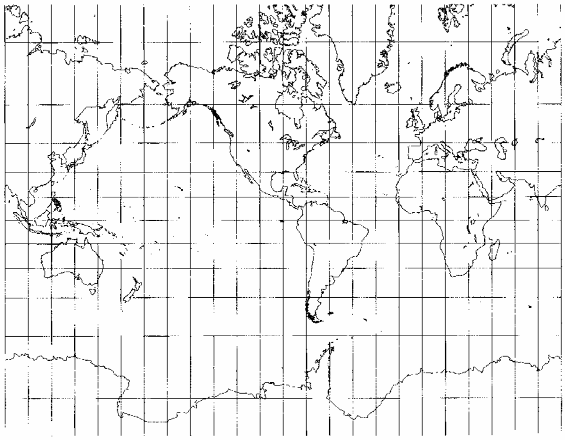
$x = a\,\lambda$

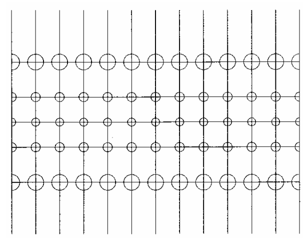
$y = ?$

$$y = a\ln\left[\tan\left(45^{\circ} + \frac{\varphi}{2}\right)\left(\frac{1-e\sin\varphi}{1+e\sin\varphi}\right)^{\frac{e}{2}}\right]$$

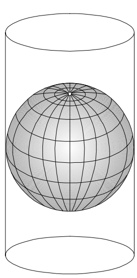
$$m_m = m_p = \frac{\sqrt{1-e^2\sin^2\varphi}}{\cos\varphi} \qquad M = \frac{1-e^2\sin^2\varphi}{\cos^2\varphi}$$

Ορθή Μερκατορική προβολή 3/3





Ορθή κυλινδρική ισοδύναμη προβολή 1/2



$$m_m\, m_p = 1 \Rightarrow \frac{\mathrm{d}s_m}{\mathrm{d}m}\frac{\mathrm{d}s_p}{\mathrm{d}p} = 1 \Rightarrow \frac{\mathrm{d}y}{R\,\mathrm{d}\varphi}\frac{\mathrm{d}x}{R\cos\varphi\,\mathrm{d}\lambda} = 1$$

$$\mathrm{d}x = R\,\mathrm{d}\lambda$$

$$\mathrm{d}y = R\cos\varphi\,\mathrm{d}\varphi \Rightarrow y = R\int_0^{\varphi}\cos\varphi\,\mathrm{d}\varphi$$

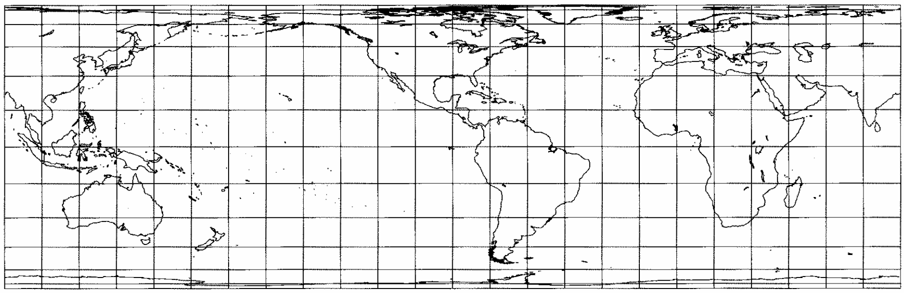
$$x = R\,\lambda$$

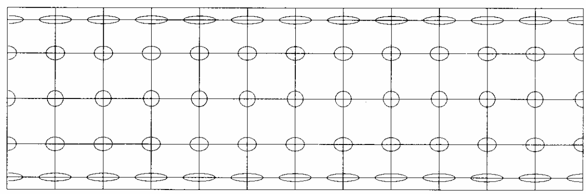
$$y = ?$$

$$y = R\sin\varphi$$

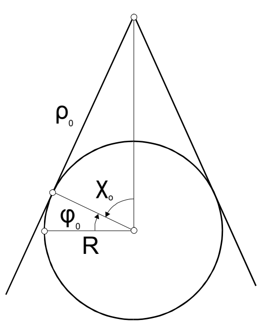
$$m_m = \cos\varphi\,,\ m_p = \frac{1}{\cos\varphi} \qquad \sin E = \frac{1-\cos^2\varphi}{1+\cos^2\varphi}$$

Ορθή κυλινδρική ισοδύναμη προβολή 2/2





Κωνικές απεικονίσεις



$$x = \rho \sin\theta,$$

$$y = \rho_0 - \rho \cos\theta,$$

$$\theta = \sin\varphi_0\left(\lambda - \bar{\lambda}\right) = \cos\chi_0\left(\lambda - \bar{\lambda}\right),$$

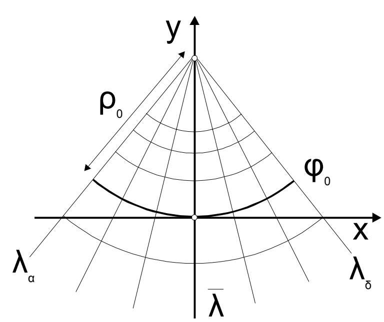
$$\bar{\lambda} = \frac{\lambda_\alpha + \lambda_\delta}{2}$$

$$\chi = 90^\circ - \varphi$$

$$\rho_0 = \frac{R}{\tan\varphi_0} = R\tan\chi_0$$

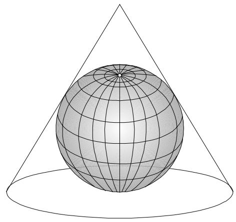
$$\mathrm{d}\theta = \frac{\mathrm{d}s_p}{\rho_0} = \frac{R\cos\varphi_0\,\mathrm{d}\lambda}{R\cot\varphi_0}$$

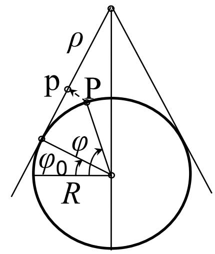
$$\theta = \sin\varphi_0\,\lambda = \cos\chi_0\,\lambda$$



Απλή κωνική προβολή (Ισαπέχουσα) 1/3

Σφαίρα





$$\theta = \sin\varphi_0 \ \lambda = \cos\chi_0 \ \lambda\,,$$

$$\rho = \rho_0 + R(\varphi_0 - \varphi)$$

$$m_m = \frac{\mathrm{d}s_m}{\mathrm{d}m} = \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi} = \frac{R\,\mathrm{d}\varphi}{R\,\mathrm{d}\varphi} = 1$$

$$m_p = \frac{\mathrm{d}s_p}{\mathrm{d}p} = \frac{\rho\,\mathrm{d}\theta}{R\cos\varphi\,\mathrm{d}\lambda} = \frac{\rho\sin\varphi_0\,\mathrm{d}\lambda}{R\cos\varphi\,\mathrm{d}\lambda} = \frac{\rho\sin\varphi_0}{R\cos\varphi}$$

Απλή κωνική προβολή (Ισαπέχουσα) 2/3

Ελλειψοειδές

$$\theta = \lambda\sin\varphi_0 = \lambda\cos\chi_0\,,$$

$$\rho = \rho_0 - \boldsymbol{M}.$$

$$\boldsymbol{M} = \mathrm{a}\,(1-e^2)\,[\boldsymbol{M}_0\varphi - \boldsymbol{M}_2\sin 2\varphi + \boldsymbol{M}_4\sin 4\varphi - \boldsymbol{M}_6\sin 6\varphi]$$

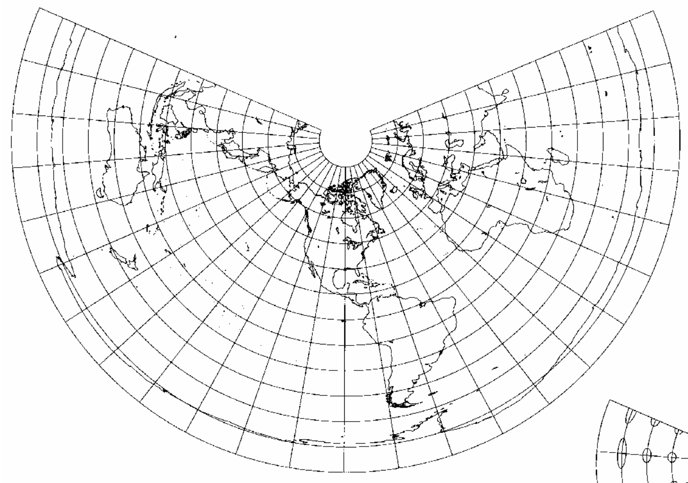
$$\boldsymbol{M}_0 = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + \frac{175}{256}e^6 + \ldots$$

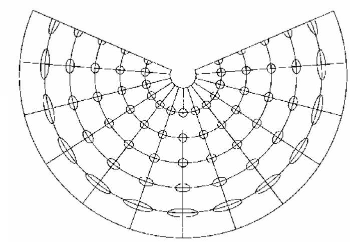
$$\boldsymbol{M}_2 = \frac{3}{8}e^2 + \frac{15}{32}e^4 + \frac{525}{1024}e^6 + \ldots$$

$$\boldsymbol{M}_4 = \frac{15}{256}e^4 + \frac{105}{1024}e^6 + \ldots$$

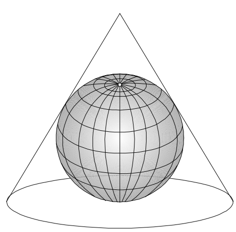
$$\boldsymbol{M}_6 = \frac{35}{3072}e^6 + \ldots$$

Απλή κωνική προβολή (Ισαπέχουσα) 3/3





Σύμμορφη κωνική προβολή (Lambert) (1/4)



$$m_m = m_p \Rightarrow \frac{\mathrm{d}s_m}{\mathrm{d}m} = \frac{\mathrm{d}s_p}{\mathrm{d}p} \Rightarrow \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi} = \frac{\rho\,\mathrm{d}\theta}{R\cos\varphi\,\mathrm{d}\lambda}$$

$$\mathrm{d}\theta = \cos\chi_0\,\mathrm{d}\lambda\,,\ \ \mathrm{d}\chi = -\mathrm{d}\varphi$$

$$\theta = \sin\varphi_0\ \lambda = \cos\chi_0\ \lambda$$

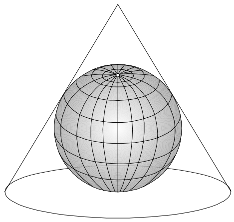
$$\rho = ?$$

$$\frac{\mathrm{d}\rho}{R\,\mathrm{d}\chi} = \frac{\rho\cos\chi_0}{R\cos\varphi} \Rightarrow \frac{\mathrm{d}\rho}{\rho} = \cos\chi_0\,\frac{\mathrm{d}\chi}{\sin\chi}$$

Σφαίρα

$$\ln\rho = \cos\chi_0\ \ln\tan\frac{\chi}{2} + C \qquad \rho = k\left[\tan\frac{\chi}{2}\right]^{\cos\chi_0}$$

Σύμμορφη κωνική προβολή (Lambert) *(2/4)*



$$\text{Για } \chi = \chi_0 : \rho = \rho_0 = R\tan\chi_0$$

$$k = R\tan\chi_0 \left[\cot\frac{\chi_0}{2}\right]^{\cos\chi_0}$$

Σφαίρα

$$\rho = R\left[\tan\chi_0 \left[\cot\frac{\chi_0}{2}\right]^{\cos\chi_0}\right]\left[\tan\frac{\chi}{2}\right]^{\cos\chi_0}$$

$$m_m = m_p = \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi}$$

Σύμμορφη κωνική προβολή (Lambert) *(3/4)*

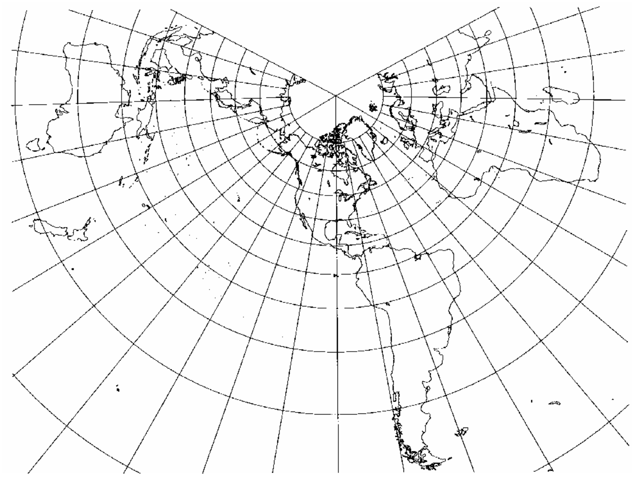
Ελλειψοειδές

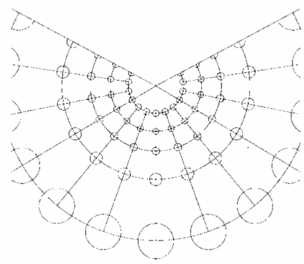
$$\theta = \lambda\sin\varphi_0$$

$$\rho = N_0\tan\chi_0 \left(\frac{\tan\left(45^\circ - \frac{\varphi}{2}\right)\left(\frac{1+e\sin\varphi}{1-e\sin\varphi}\right)^{\frac{e}{2}}}{\tan\left(45^\circ - \frac{\varphi_0}{2}\right)\left(\frac{1+e\sin\varphi_0}{1-e\sin\varphi_0}\right)^{\frac{e}{2}}} \right)^{\sin\varphi_0}$$

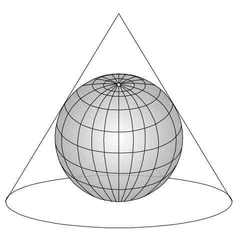
$$m = m_\mathrm{m} = m_\mathrm{p} = \frac{\rho\sin\varphi_0}{N\cos\varphi}$$

Σύμμορφη κωνική προβολή (Lambert) (4/4)





Ισοδύναμη κωνική προβολή (Albers) 1/2



$$m_m\, m_p = 1 \Rightarrow \frac{\mathrm{d}s_m}{\mathrm{d}m}\frac{\mathrm{d}s_p}{\mathrm{d}p} = 1 \Rightarrow \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi}\frac{\rho\,\mathrm{d}\theta}{R\cos\varphi\,\mathrm{d}\lambda} = 1$$

$$\mathrm{d}\theta = \cos\chi_0\,\mathrm{d}\lambda\,,\ \mathrm{d}\chi = -\mathrm{d}\varphi$$

$\theta = \sin\varphi_0\,\lambda = \cos\chi_0\,\lambda$

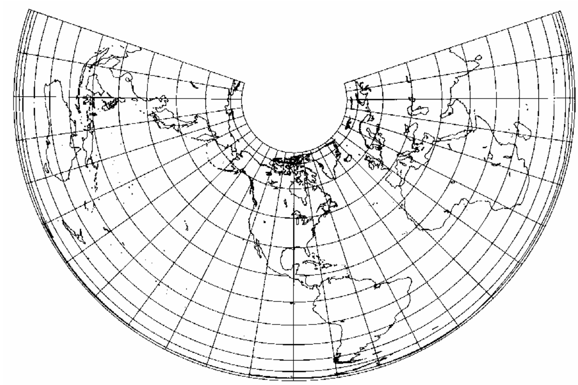
$\rho = ?$

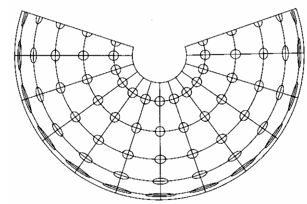
$$\frac{\mathrm{d}\rho}{R\,\mathrm{d}\chi}\frac{\rho\cos\chi_0}{R\sin\chi} = 1 \Rightarrow \rho\,\mathrm{d}\rho = R^2\frac{\sin\chi}{\cos\chi_0}\mathrm{d}\chi$$

$$\rho^2 = -2R^2\frac{\cos\chi}{\cos\chi_0} + C \qquad C = R^2\tan^2\chi_0 + 2R^2$$

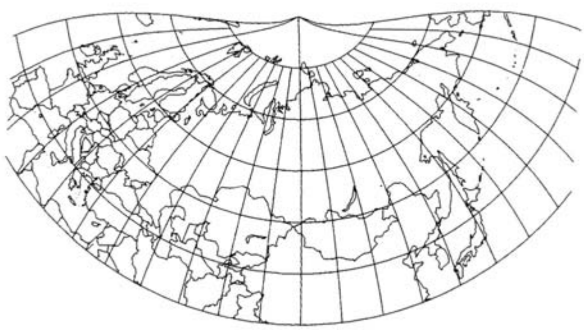
$$\rho^2 = R^2\tan^2\chi_0 + 2R^2\left[1 - \frac{\cos\chi}{\cos\chi_0}\right]$$

Ισοδύναμη κωνική προβολή (Albers) 2/2





Ισοδύναμη προβολή Bonne

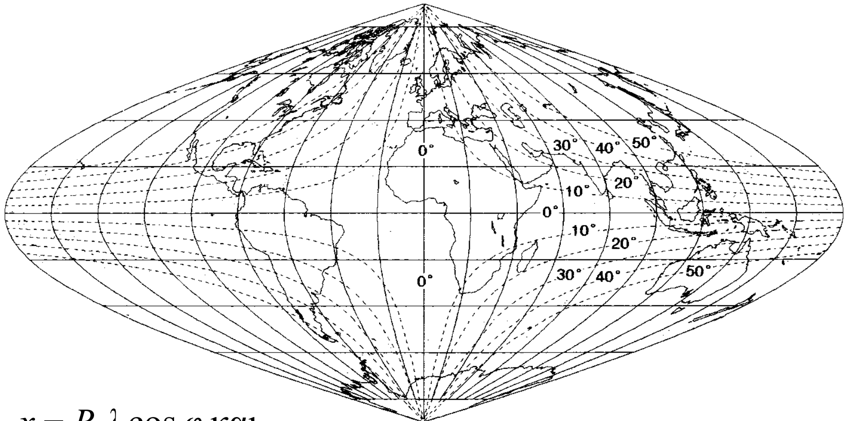


$$dp = R\cos\varphi\, d\lambda = R\sin\chi\, d\lambda$$

$$d\theta = \frac{ds_p}{\rho} = \frac{R\sin\chi}{\rho} d\lambda = \frac{R\cos\varphi}{\rho} d\lambda \Rightarrow \theta = \frac{R\sin\chi}{\rho}\lambda = \frac{R\cos\varphi}{\rho}\lambda$$

$$\rho = \rho_0 + R\left(\varphi_0 - \varphi\right) = \rho_0 + R\left(\chi - \chi_0\right) \qquad \rho_0 = \frac{R}{\tan\varphi_0} = R\tan\chi_0$$

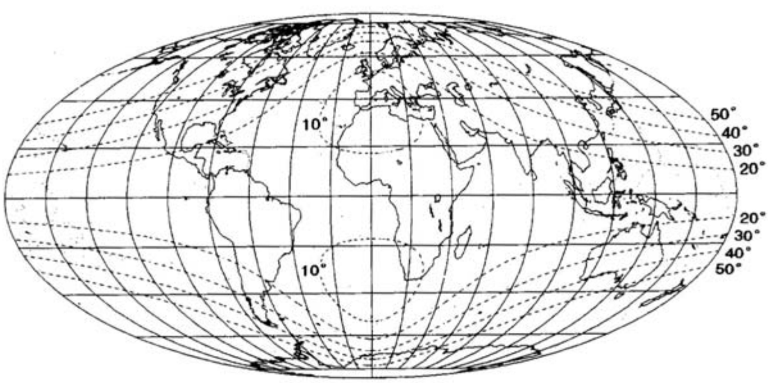
Ισοδύναμη προβολή Sanson



$$x = R\,\lambda \cos\varphi \text{ και}$$

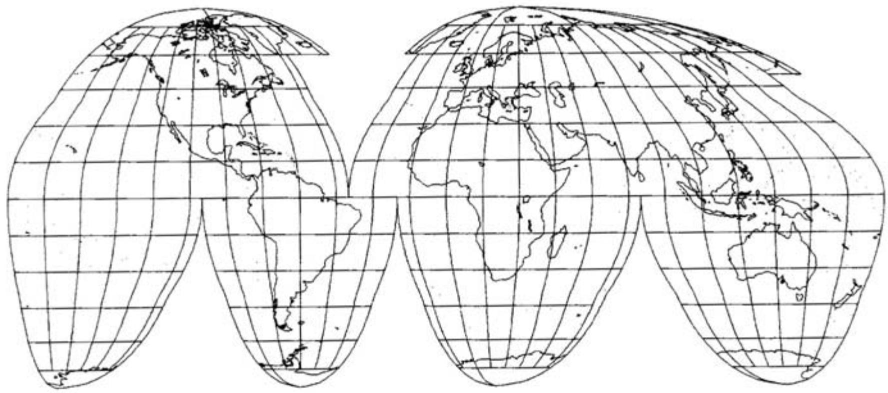
$$y = R\,\varphi.$$

Ισοδύναμη προβολή Mollweide

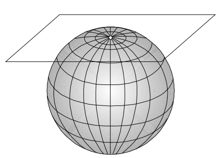


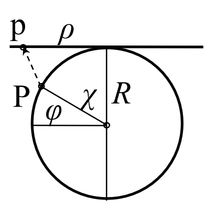
$$y = R\sqrt{2}\,\sin\omega \qquad \sin\varphi = \frac{2\omega}{\pi} + \frac{\sin 2\omega}{\pi}$$

Παράδειγμα διακεκομμένης προβολής



Ορθή επίπεδη ισαπέχουσα προβολή (Postel) *1/2*





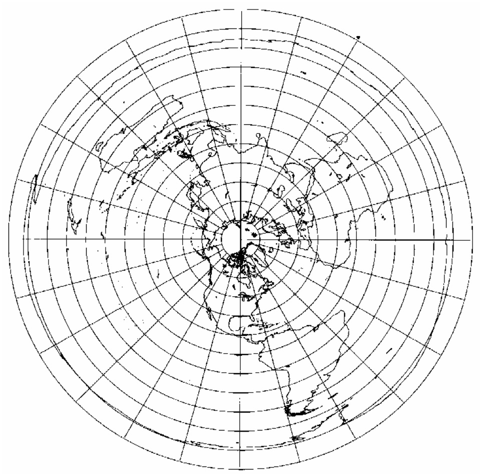
$$\theta = \lambda, \qquad \rho = R\,\chi$$

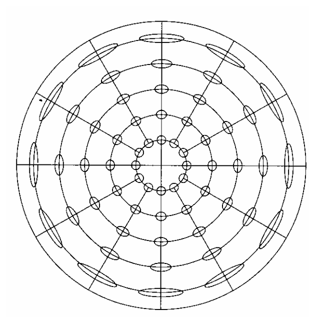
$$m_m = \frac{\mathrm{d}s_m}{\mathrm{d}m} = \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi} = \frac{-R\,\mathrm{d}\chi}{-R\,\mathrm{d}\chi} = 1$$

$$m_p = \frac{\mathrm{d}s_p}{\mathrm{d}p} = \frac{\rho\,\mathrm{d}\theta}{R\cos\varphi\,\mathrm{d}\lambda} = \frac{R\,\chi\,\mathrm{d}\lambda}{R\sin\chi\,\mathrm{d}\lambda} = \frac{\chi}{\sin\chi}$$

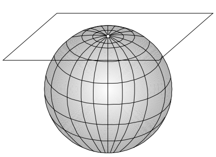
$$M = \frac{\chi}{\sin\chi} \qquad \sin E = \frac{\chi - \sin\chi}{\chi + \sin\chi}$$

Ορθή επίπεδη ισαπέχουσα προβολή (Postel) 2/2





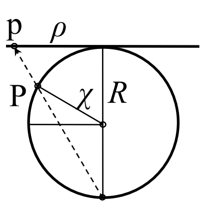
Πολική στερεογραφική προβολή 1/3



$\theta = \lambda,$

$\rho = ?$

$$m_m = m_p \Rightarrow \frac{\mathrm{d}s_m}{\mathrm{d}m} = \frac{\mathrm{d}s_p}{\mathrm{d}p} \Rightarrow \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi} = \frac{\rho\,\mathrm{d}\theta}{R\cos\varphi\,\mathrm{d}\lambda}$$



$$\mathrm{d}\theta = \mathrm{d}\lambda\,,\ \mathrm{d}\chi = -\mathrm{d}\varphi$$

$$\frac{\mathrm{d}\rho}{R\,\mathrm{d}\chi} = \frac{\rho}{R\cos\varphi} \Rightarrow \frac{\mathrm{d}\rho}{\rho} = \frac{\mathrm{d}\chi}{\sin\chi}$$

$$\ln\rho = \ln\tan\frac{\chi}{2} + C \Rightarrow \rho = c\tan\frac{\chi}{2}$$

Σφαίρα

$$\rho = 2R\tan\frac{\chi}{2}$$

$$m_m = m_p = \frac{1}{\cos^2\frac{\chi}{2}} \qquad M = \frac{1}{\cos^4\frac{\chi}{2}}$$

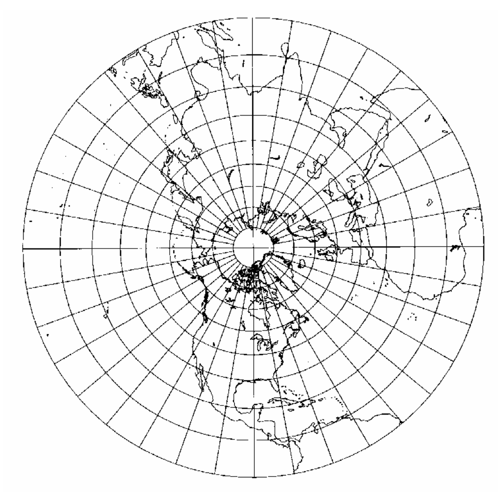
Πολική στερεογραφική προβολή 2/3

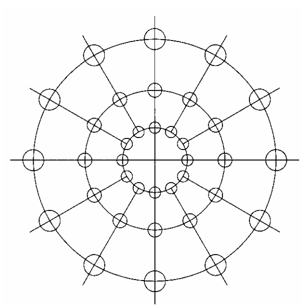
Ελλειψοειδές

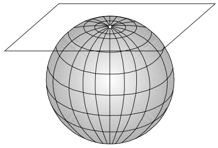
$$\theta = \lambda,$$

$$\rho = \left(\frac{2a^2}{b} \left(\frac{1-e}{1+e} \right)^{\frac{e}{2}} \right) \tan\left(45^\circ - \frac{\varphi}{2} \right) \left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi} \right)^{\frac{e}{2}}.$$

Πολική στερεογραφική προβολή 3/3







$\theta = \lambda,$

$\rho = ?$

$$m_m\, m_p = 1 \Rightarrow \frac{ds_m}{dm}\frac{ds_p}{dp} = 1 \Rightarrow \frac{-d\rho}{R\,d\varphi}\frac{\rho\,d\theta}{R\cos\varphi\,d\lambda} = 1$$

$$d\theta = d\lambda\,,\ d\chi = -d\varphi$$

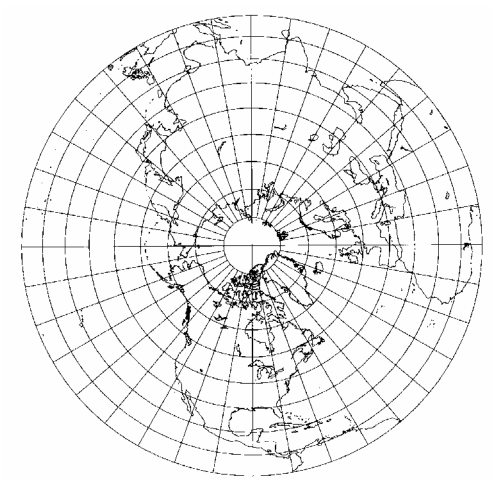
$$\frac{d\rho}{R\,d\chi}\frac{\rho}{R\cos\varphi} = 1 \Rightarrow \rho\,d\rho = R^2 \sin\chi\,d\chi$$

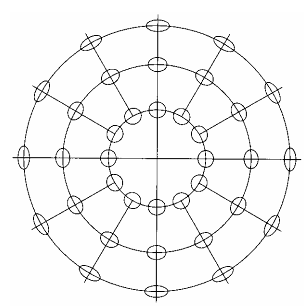
$$\rho^2 = -2R^2\cos\chi + C \Rightarrow \rho^2 = 4R^2\sin^2\frac{\chi}{2} - 2R^2 + c$$

Για $\chi = 0 : \rho = 0 \Rightarrow c = 2R^2$

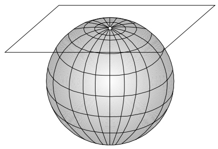
$$\rho = 2R\sin\frac{\chi}{2}$$

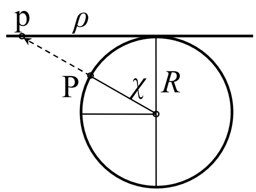
$$m_m = \cos\frac{\chi}{2}\ ,\ m_p = \frac{1}{\cos\frac{\chi}{2}} \qquad \sin E = \frac{1-\cos^2\frac{\chi}{2}}{1+\cos^2\frac{\chi}{2}}$$





Πολική γνωμονική προβολή 1/2





$\theta = \lambda,$

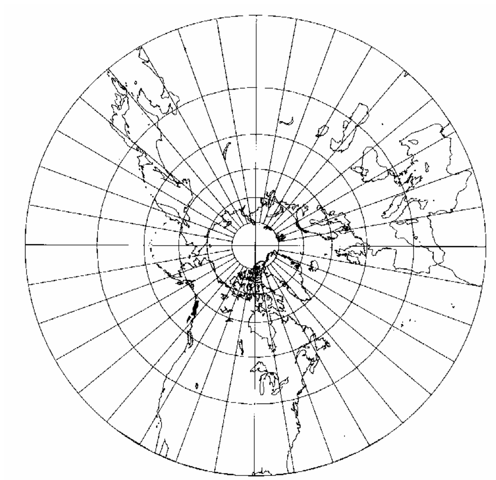
$\rho = R \tan \chi$

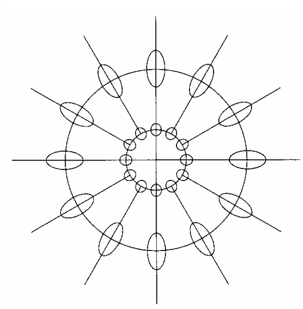
$$m_m = \frac{\mathrm{d}s_m}{\mathrm{d}m} = \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi} = \frac{-\dfrac{R}{\cos^2 \chi}\mathrm{d}\chi}{-R\,\mathrm{d}\chi} = \frac{1}{\cos^2 \chi}$$

$$m_p = \frac{\mathrm{d}s_p}{\mathrm{d}p} = \frac{\rho\,\mathrm{d}\theta}{R\cos\varphi\,\mathrm{d}\lambda} = \frac{R\tan\chi\,\mathrm{d}\lambda}{R\sin\chi\,\mathrm{d}\lambda} = \frac{1}{\cos\chi}$$

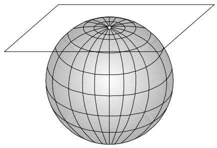
$$M = \frac{1}{\cos^3 \chi} \qquad \sin E = \tan^2 \frac{\chi}{2}$$

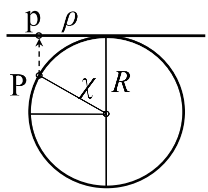
Πολική γνωμονική προβολή 2/2





Πολική ορθογραφική προβολή 1/2





$$\theta = \lambda,$$

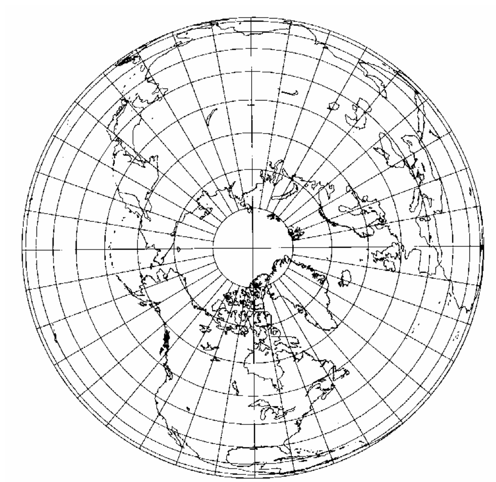
$$\rho = R \sin \chi$$

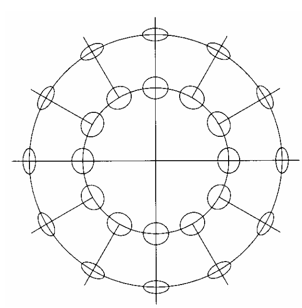
$$m_m = \frac{\mathrm{d}s_m}{\mathrm{d}m} = \frac{-\mathrm{d}\rho}{R\,\mathrm{d}\varphi} = \frac{R \cos \chi \,\mathrm{d}\chi}{R\,\mathrm{d}\chi} = \cos \chi$$

$$m_p = \frac{\mathrm{d}s_p}{\mathrm{d}p} = \frac{\rho\,\mathrm{d}\theta}{R \cos\varphi\,\mathrm{d}\lambda} = \frac{R \sin \chi \,\mathrm{d}\lambda}{R \sin \chi \,\mathrm{d}\lambda} = 1$$

$$M = \cos \chi \qquad \sin E = \tan^2 \frac{\chi}{2}$$

Πολική ορθογραφική προβολή 2/2





Συγκριτική παρουσίαση πολικών απεικονίσεων