

Άσκηση 10^η

Παρεμβολή – προσαρμογή καμπυλών

$$\begin{aligned}
 a_0 \sum_{i=0}^N x_i^0 + a_1 \sum_{i=0}^N x_i^1 + a_2 \sum_{i=0}^N x_i^2 + \dots + a_m \sum_{i=0}^N x_i^m &= \sum_{i=0}^N f_i, \\
 a_0 \sum_{i=0}^N x_i^1 + a_1 \sum_{i=0}^N x_i^2 + a_2 \sum_{i=0}^N x_i^3 + \dots + a_m \sum_{i=0}^N x_i^{m+1} &= \sum_{i=0}^N x_i f_i, \\
 a_0 \sum_{i=0}^N x_i^2 + a_1 \sum_{i=0}^N x_i^3 + a_2 \sum_{i=0}^N x_i^4 + \dots + a_m \sum_{i=0}^N x_i^{m+2} &= \sum_{i=0}^N x_i^2 f_i, \\
 \dots & \\
 a_0 \sum_{i=0}^N x_i^m + a_1 \sum_{i=0}^N x_i^{m+1} + a_2 \sum_{i=0}^N x_i^{m+2} + \dots + a_m \sum_{i=0}^N x_i^{2m} &= \sum_{i=0}^N x_i^m f_i.
 \end{aligned}$$

ΚΡΙΤΗΡΙΟ : $E(p) = \sum_{i=0}^N (p(x_i) - f_i)^2 \longrightarrow \min$

$$y = f(x) = p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$N = 6 \quad m = 3$$

x	y
0	-14,50
1	0,20
2	3,10
3	-0,50
4	2,80
5	0,10
6	16,00

$$\begin{pmatrix} \sum x^0 & \sum x & \sum x^2 & \sum x^3 \\ \sum x & \sum x^2 & \sum x^3 & \sum x^4 \\ \sum x^2 & \sum x^3 & \sum x^4 & \sum x^5 \\ \sum x^3 & \sum x^4 & \sum x^5 & \sum x^6 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \sum y \\ \sum xy \\ \sum x^2 y \\ \sum x^3 y \end{pmatrix}$$

x	y
0	-14,50
1	0,20
2	3,10
3	-0,50
4	2,80
5	0,10
6	16,00

$$\Sigma x^0 = 7$$

$$\Sigma y = 7,2$$

$$\Sigma x = 21$$

$$\Sigma xy = 112,6$$

$$\Sigma x^2 = 91$$

$$\Sigma x^2 y = 631,4$$

$$\Sigma x^3 = 441$$

$$\Sigma x^3 y = 3659,2$$

$$\Sigma x^4 = 2275$$

$$\Sigma x^5 = 12201$$

$$\Sigma x^6 = 67171$$

$$\begin{pmatrix} 7 & 21 & 91 & 441 \\ 21 & 91 & 441 & 2275 \\ 91 & 441 & 2275 & 12201 \\ 441 & 2275 & 12201 & 67171 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 7,2 \\ 112,6 \\ 631,4 \\ 3659,2 \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 7 & 21 & 91 & 441 \\ 21 & 91 & 441 & 2275 \\ 91 & 441 & 2275 & 12201 \\ 441 & 2275 & 12201 & 67171 \end{pmatrix}^{-1} \begin{pmatrix} 7,2 \\ 112,6 \\ 631,4 \\ 3659,2 \end{pmatrix}$$

$$\begin{aligned} a_0 &= -14,360 \\ a_1 &= 21,002 \\ a_2 &= -7,823 \\ a_3 &= 0,858 \end{aligned}$$

$$s_y = \pm \sqrt{\frac{\sum_{i=1}^{N+1} (y_i - \tilde{y}_i)^2}{N+1}} = \sqrt{\frac{20,920603}{7}} = \pm 1,73$$