Τι είναι το φίλτρο

Τι είδη φίλτρων έχουμε ;

Ένα ψηφιακό φίλτρο ορίζεται ως η υπολογιστική διαδικασία με τη βοήθεια της οποίας ένα διακριτό σήμα, δηλαδή μια ακολουθία αριθμών, μετασχηματίζεται σε μια δεύτερη ακολουθία αριθμών που εκφράζουν το σήμα εξόδου (J.F. Kaiser).

Στην επεξεργασία σήματος, η λειτουργία ενός φίλτρου απομακρύνει τα ανεπιθύμητα μέρη ενός σήματος, όπως έναν τυχαίο θόρυβο, ή εξάγει χρήσιμα κομμάτια ενός σήματος, όπως οι συνιστώσες που βρίσκονται σε μια συγκεκριμένη περιοχή συχνοτήτων.

Είδη Φίλτρων

- Παθητικά φίλτρα
- Ενεργητικά φίλτρα

Είδη Φίλτρων

- Αναλογικά Φίλτρα
- Ψηφιακά φίλτρα :

Συνέλιξη με κρουστική απόκριση φίλτρου

⇩

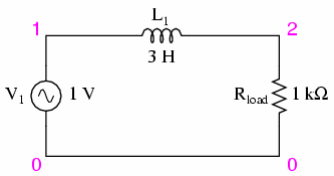
Kernel filter (φίλτρα FIR)

ή

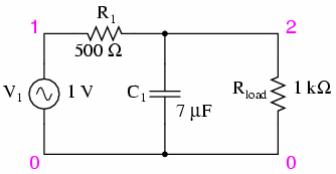
Ορισμός αναδρομικών συντελεστών (IIR)

Example of a Low Pass Filter

Inductive low-pass filter



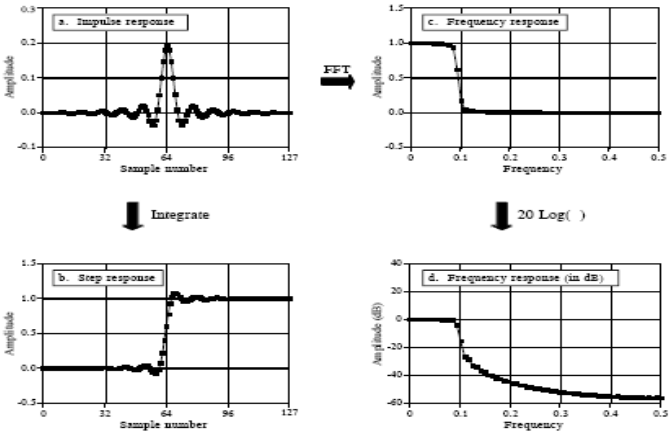
Capacitive low-pass filter




```
inductive lowpass filter
v1 1 0 ac 1 sin
l1 1 2 3
rload 2 0 1k
.ac lin 20 1 200
.plot ac v(2)
.end
freq      v(2)  0.2512    0.3981    0.631     1
-------------------------------------------------
1.000E+00 9.998E-01 .       .         .         *
1.147E+01 9.774E-01 .       .         .         *.
2.195E+01 9.240E-01 .       .         .        *.
3.242E+01 8.533E-01 .       .         .      *  .
4.289E+01 7.776E-01 .       .         .     *   .
5.337E+01 7.050E-01 .       .         .   *     .
6.384E+01 6.391E-01 .       .         .  *      .
7.432E+01 5.810E-01 .       .        *  .      .
8.479E+01 5.304E-01 .       .       *    .      .
9.526E+01 4.865E-01 .       .     *      .      .
1.057E+02 4.485E-01 .       .   *        .      .
1.162E+02 4.153E-01 .       .*           .      .
1.267E+02 3.863E-01 .       *            .      .
1.372E+02 3.607E-01 .     *              .      .
1.476E+02 3.382E-01 .   *                .      .
1.581E+02 3.181E-01 . *                  .      .
1.686E+02 3.002E-01 .*                   .      .
1.791E+02 2.841E-01 . *                  .      .
1.895E+02 2.696E-01 .*                   .      .
2.000E+02 2.564E-01 .*                   .      .
-------------------------------------------------
```

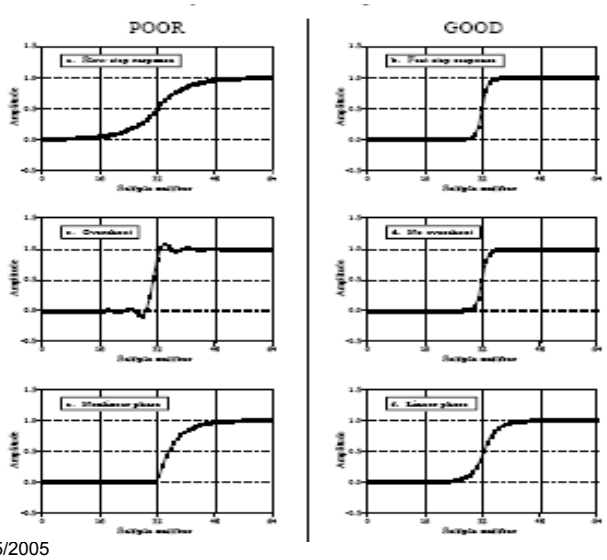
Load voltage decreases with increasing frequency

Παράμετροι Φίλτρων

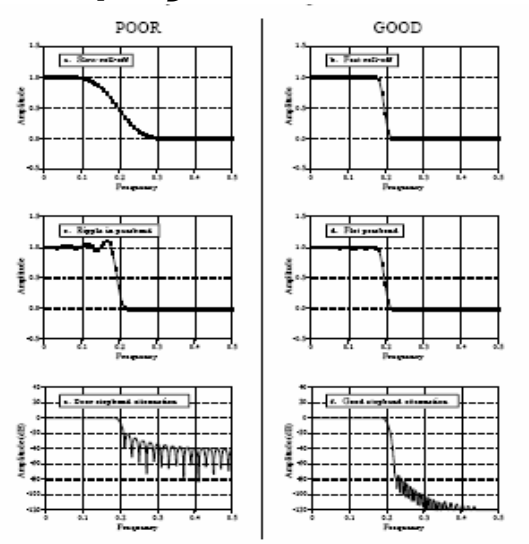


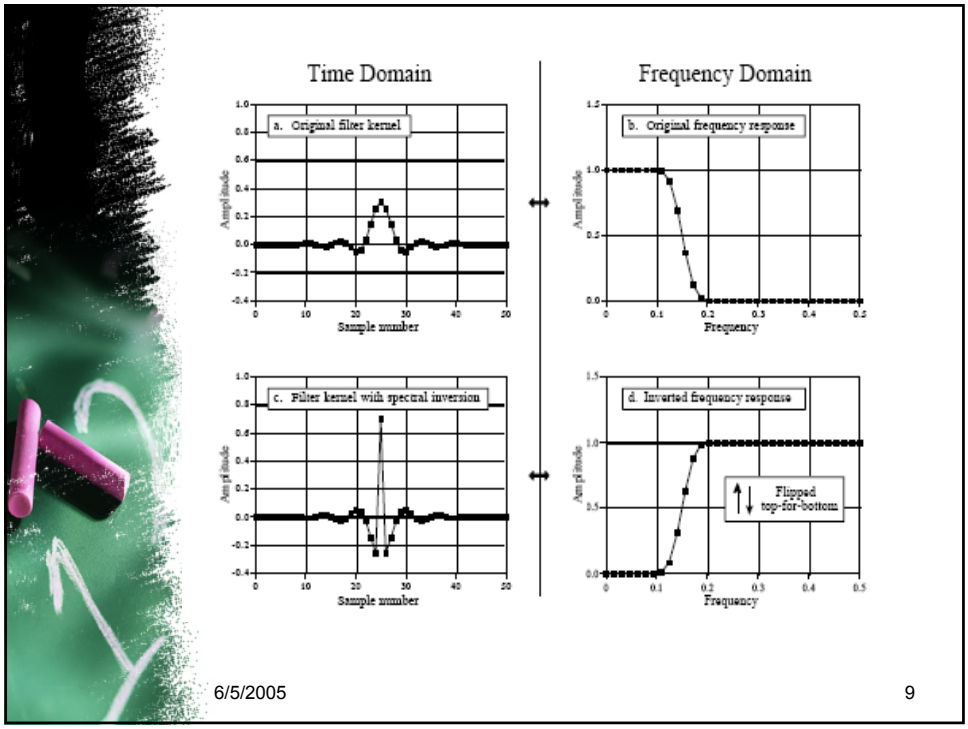
Kernel Filter

Παράμετροι στο πεδίο του χρόνου



Παράμετροι στο πεδίο της συχνότητας





Τι είδη φίλτρων έχουμε;

- Low Pass -Χαμηλοπερατά φίλτρα
- High Pass -Υψηλοπερατά φίλτρα
- Band Pass -Φίλτρα εύρους ζώνης
- Band Stop -Φίλτρα φραγμού ζώνης
- Resonant filters

Τι είδη φίλτρων έχουμε;

Χαμηλοπερατά φίλτρα

$$|H(j\Omega)| = \begin{cases} 1 \text{ for } \Omega \in [-\Omega_p, \Omega_p] \\ 0 \text{ otherwise} \end{cases}$$

Υψηλοπερατά φίλτρα

$$|H(j\Omega)| = \begin{cases} 0 \text{ for } \Omega \in [-\Omega_p, \Omega_p] \\ 1 \text{ otherwise} \end{cases}$$

Τι είδη φίλτρων έχουμε ;

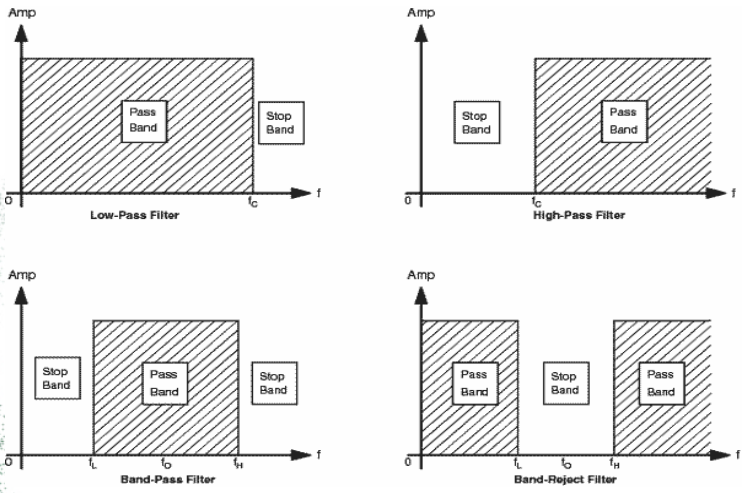
Φίλτρα εύρους ζώνης

$$|H(j\Omega)| = \begin{cases} 1 \text{ for } \Omega \in [-\Omega_H, -\Omega_L] \text{ or } \Omega \in [\Omega_L, \Omega_H] \\ 0 \text{ otherwise} \end{cases}$$

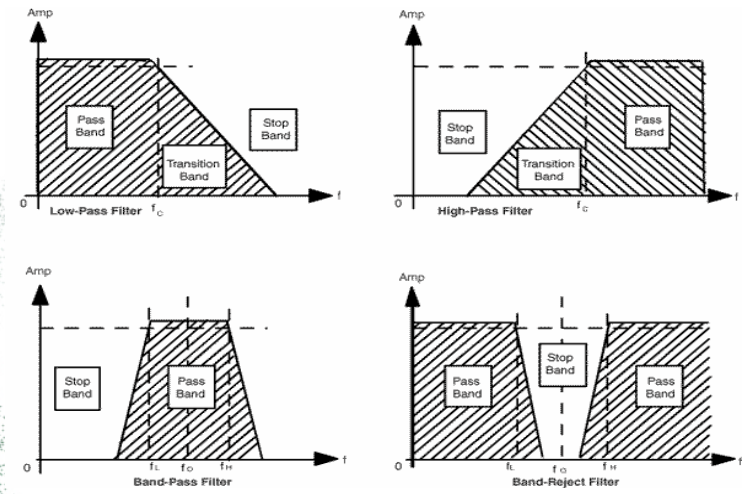
Φίλτρα φραγμού ζώνης

$$|H(j\Omega)| = \begin{cases} 0 \text{ for } \Omega \in [-\Omega_H, -\Omega_L] \text{ or } \Omega \in [\Omega_L, \Omega_H] \\ 1 \text{ otherwise} \end{cases}$$

Συμπεράσματα-Τα ιδεατά φίλτρα

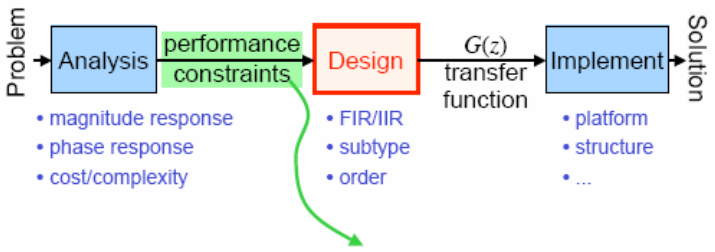


NON-IDEAL FILTERS (REAL WORLD)



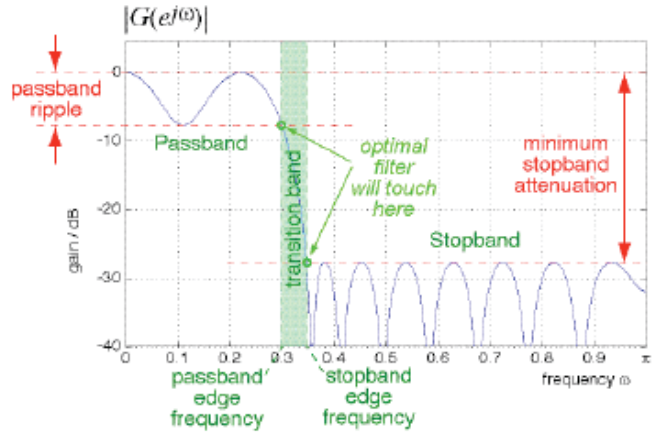
Filter Design Specifications

- The filter design process:



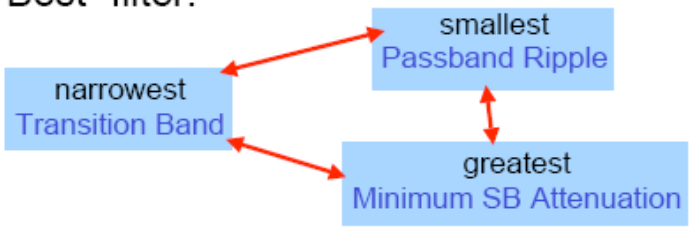
Performance Constraints

- .. in terms of magnitude response:



Performance Constraints

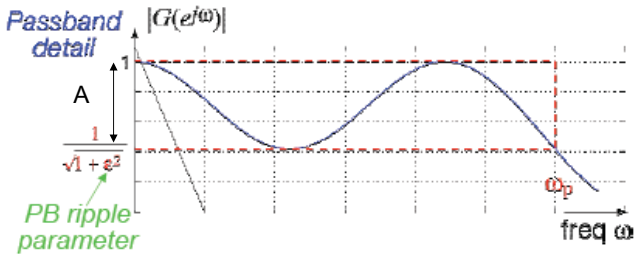
- "Best" filter:



  - improving one usually worsens others
- But: increasing filter order (i.e. cost) improves all three measures

Filter order: Αριθμός προηγούμενων εισόδων απαιτούμενος για την τρέχουσα εξαγόμενη τιμή.

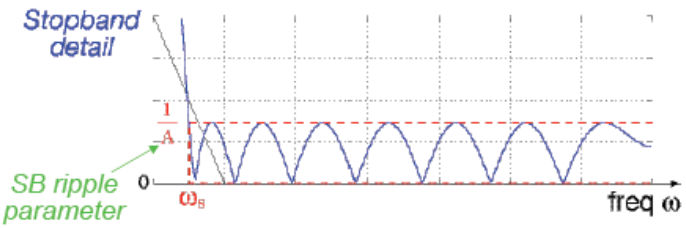
Passband Ripple



- Assume peak passband gain = 1 then *minimum* passband gain = $\frac{1}{\sqrt{1+\varepsilon^2}}$

Μικρό ε ελαττώνει το πλάτος της ανεπιθύμητης ταλάντωσης (ripple)

Stopband Ripple



- Peak passband gain is $A\times$ larger than peak stopband gain
- Hence, minimum stopband attenuation

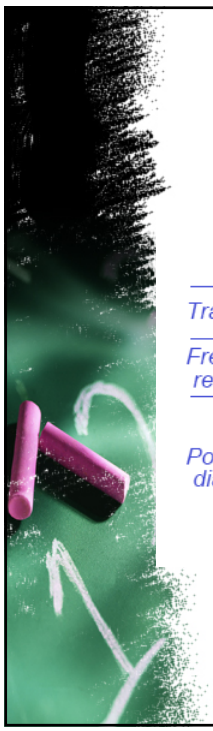
$$\alpha_s = -20\log_{10}\tfrac{1}{A} = 20\log_{10} A \quad \text{dB}$$

Filter Type Choice: FIR vs. IIR

	FIR	IIR
	No feedback (just zeros)	Feedback (poles & zeros)
	Always stable	May be unstable
	Can be linear phase	Difficult to control phase
BUT	High order (20-2000)	Typ. < 1/10th order of FIR (4-20)
	Unrelated to continuous-time filtering	Derive from *analog prototype*

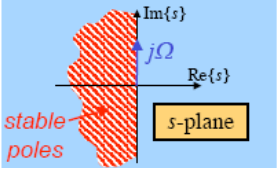
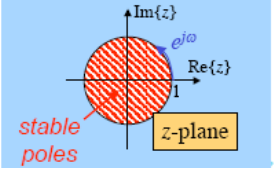
FIR vs. IIR

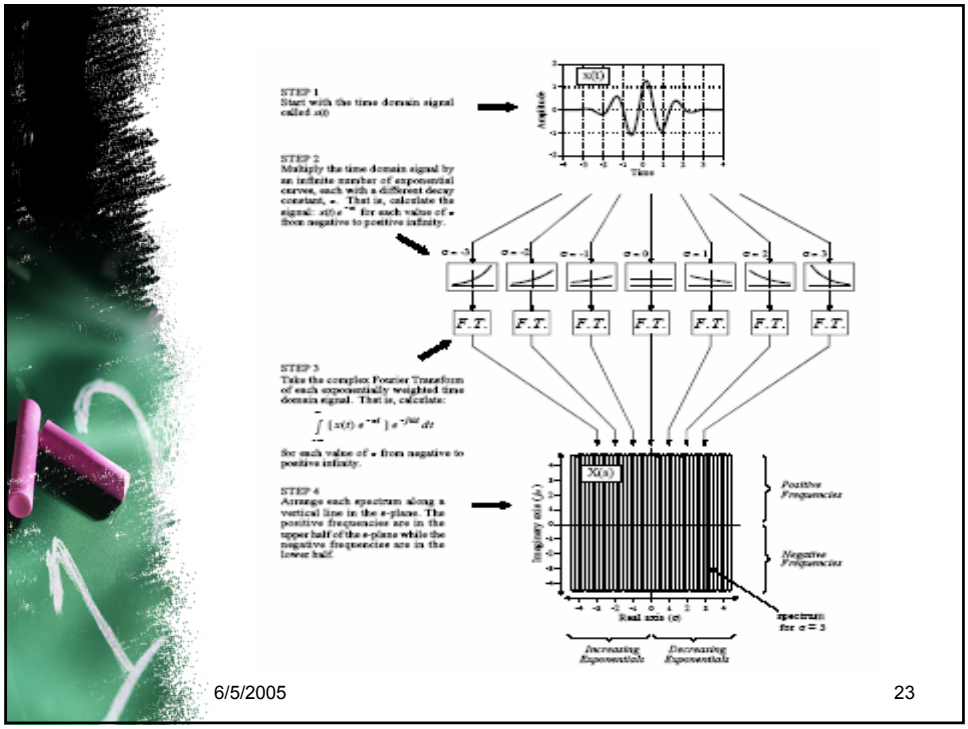
- If you care about computational cost
  → use low-complexity IIR
  (computation no object → Lin Phs FIR)
- If you care about phase response
  → use linear-phase FIR
  (phase unimportant → go with simple IIR)

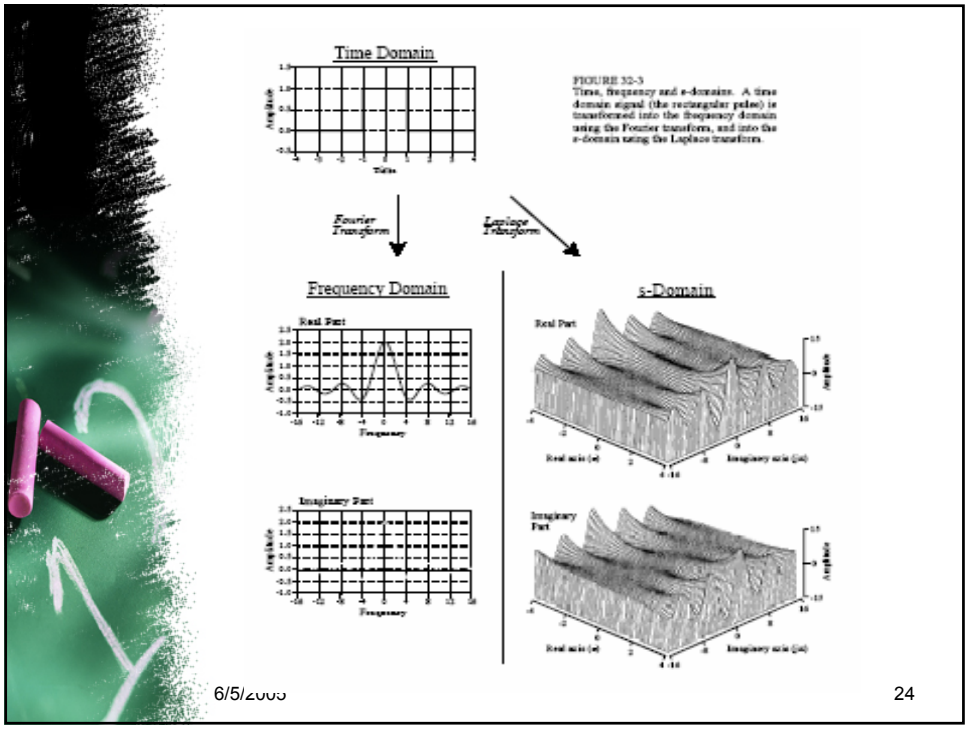


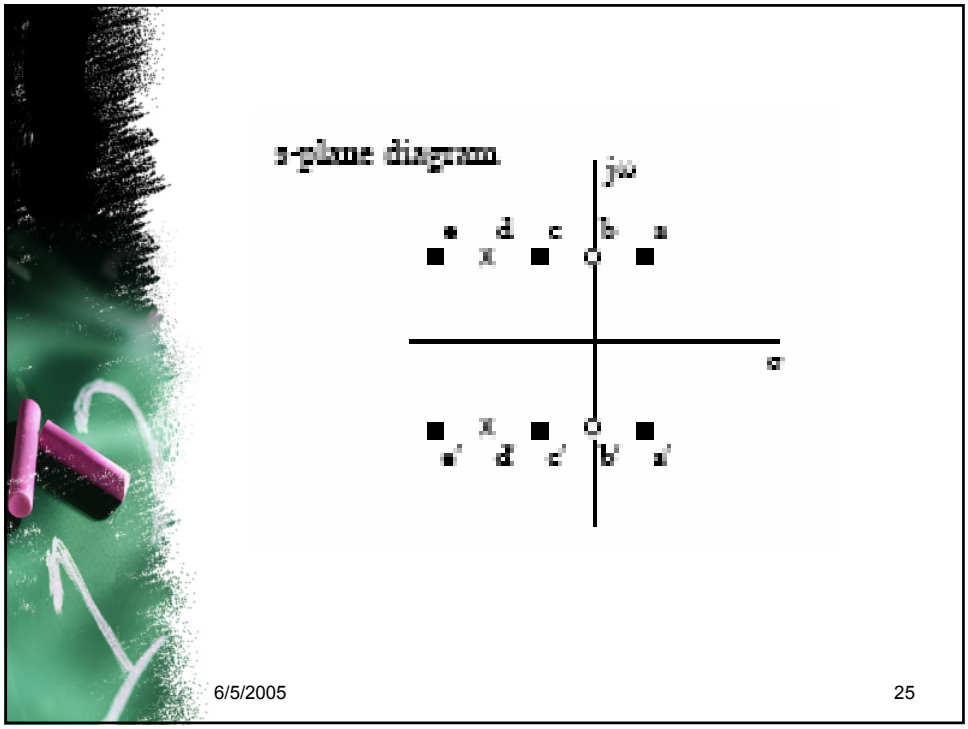
CT Transfer Functions

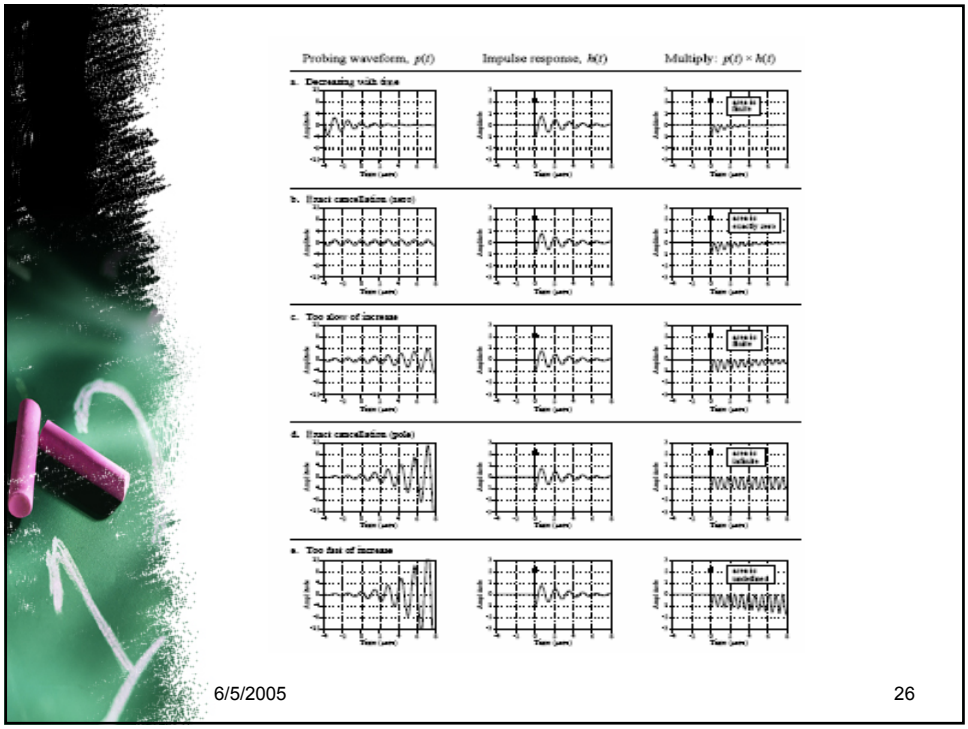
- Analog systems: s-transform (Laplace)

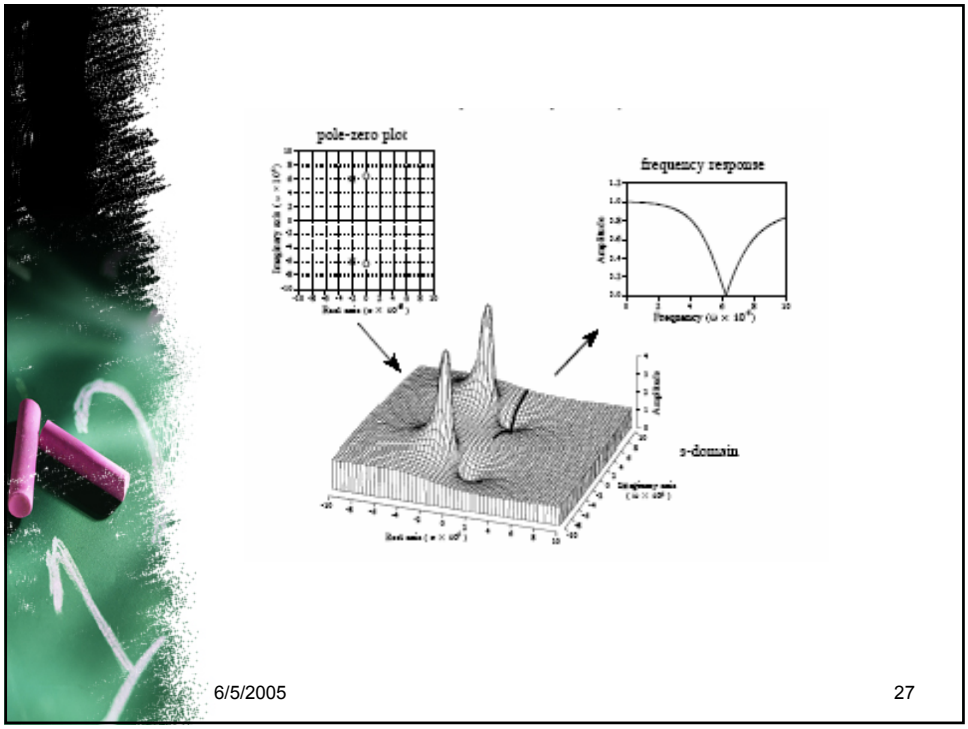
	Continuous-time	Discrete-time
Transform	$H_a(s) = \int h_a(t) e^{-st} dt$	$H_d(z) = \sum h_d[n] z^{-n}$
Frequency response	$H_a(j\Omega)$	$H_d(e^{j\omega})$
Pole/zero diagram		

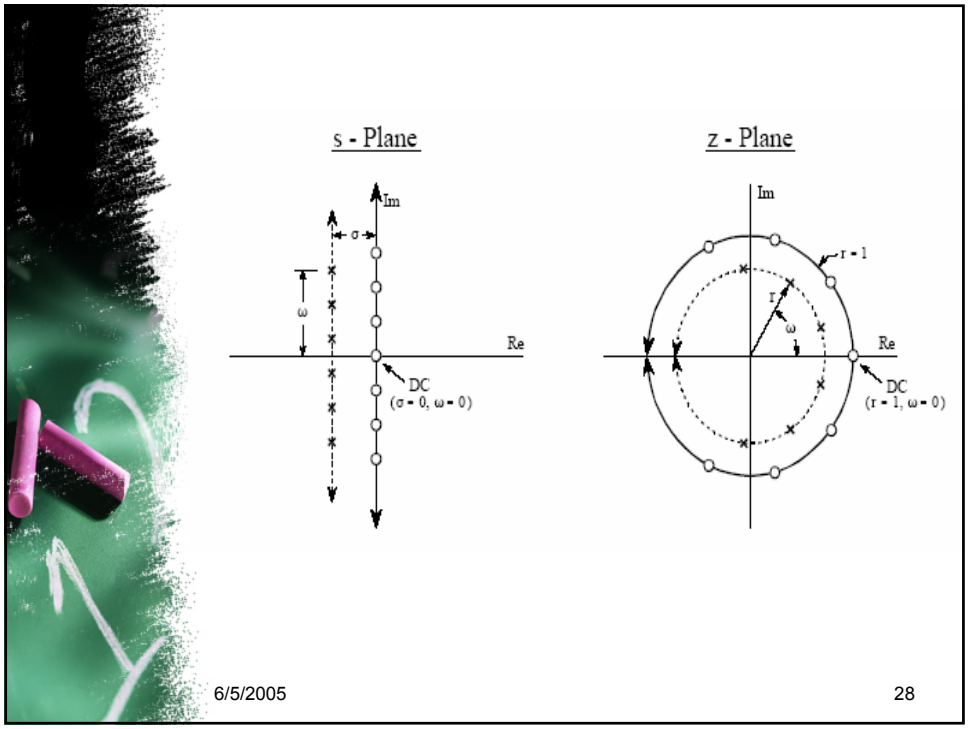

STEP 1
Start with the time domain signal called x(t)
STEP 2
Multiply the time domain signal by an infinite number of exponential curves, each with a different decay constant, σ. That is, calculate the signal: x(t)e^{-σt} for each value of σ from negative to positive infinity.
STEP 3
Take the complex Fourier Transform of each exponentially weighted time domain signal. That is, calculate:
for each value of σ from negative to positive infinity.
STEP 4
Arrange each spectrum along a vertical line in the s-plane. The positive frequencies are in the upper half of the s-plane while the negative frequencies are in the lower half.
x(t)
Amplitude
Time
σ = -3
σ = -2
σ = -1
σ = 0
σ = 1
σ = 2
σ = 3
F.T.
X(s)
Positive Frequencies
Negative Frequencies
Imaginary axis (jω)
Real axis (σ)
spectrum for σ = 3
Increasing Exponentials
Decreasing Exponentials


Time Domain
Amplitude
Time
FIGURE 32-3
Time, frequency and s-domains. A time domain signal (the rectangular pulse) is transformed into the frequency domain using the Fourier transform, and into the s-domain using the Laplace transform.
Fourier Transform
Laplace Transform
Frequency Domain
Real Part
Amplitude
Frequency
Imaginary Part
s-Domain
Real Part
Real axis (σ)
Imaginary axis (jω)
Imaginary Part
Amplitude


s-plane diagram
jω
d
c
b
a
σ
a'
d'
c'
b'
a'


Probing waveform, p(t)
Impulse response, h(t)
Multiply: p(t) × h(t)
a. Decreasing with time
b. Exact cancellation (zero)
c. Too slow of increase
d. Exact cancellation (pole)
e. Too fast of increase

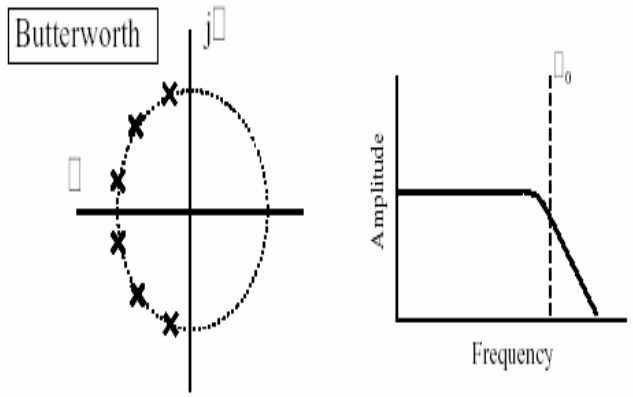

pole-zero plot
frequency response
s-domain


s - Plane
Im
σ
ω
Re
DC
(σ = 0, ω = 0)
z - Plane
Im
r = 1
r
ω
Re
DC
(r = 1, ω = 0)

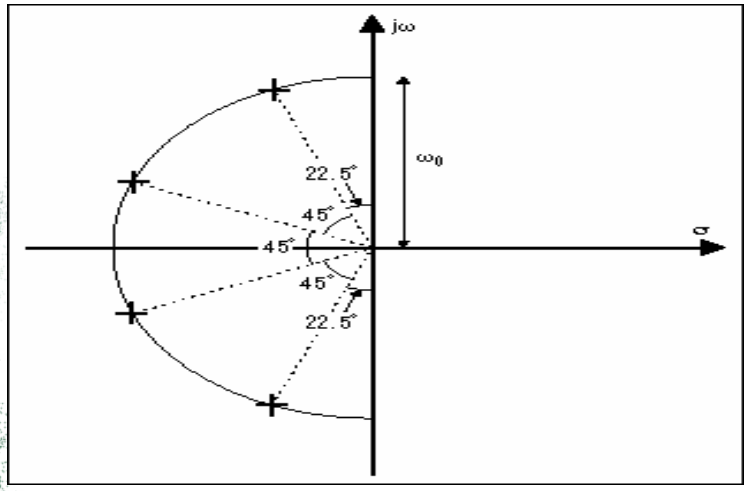
Παραδείγματα φίλτρων

- Butterworth
- Chebyshev
- Elliptic
- Bessel

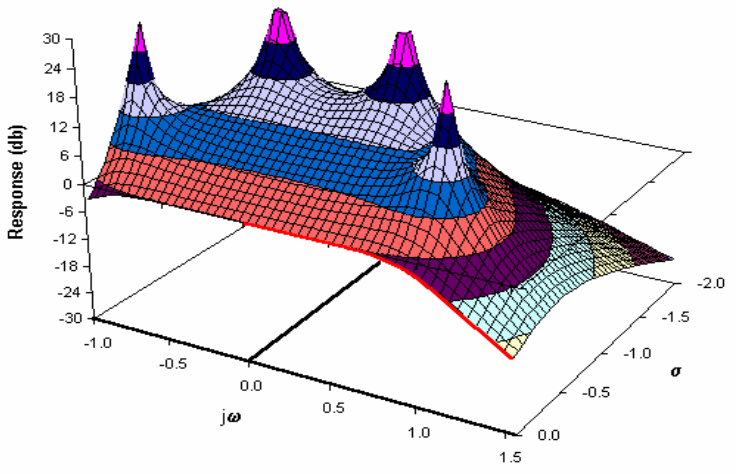
A low-pass Butterworth filter is designed



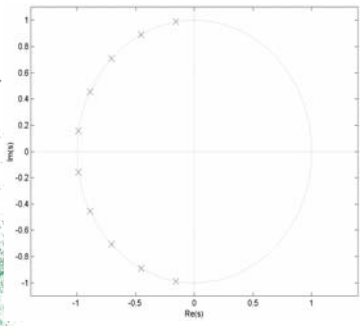
A low-pass Butterworth filter is designed(4 poles)

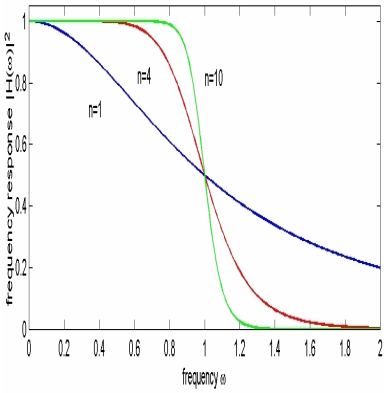


A low-pass Butterworth filter is designed(4 poles)



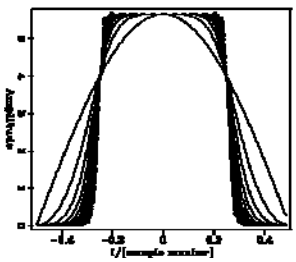
Butterworth filter

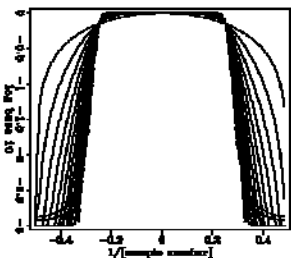




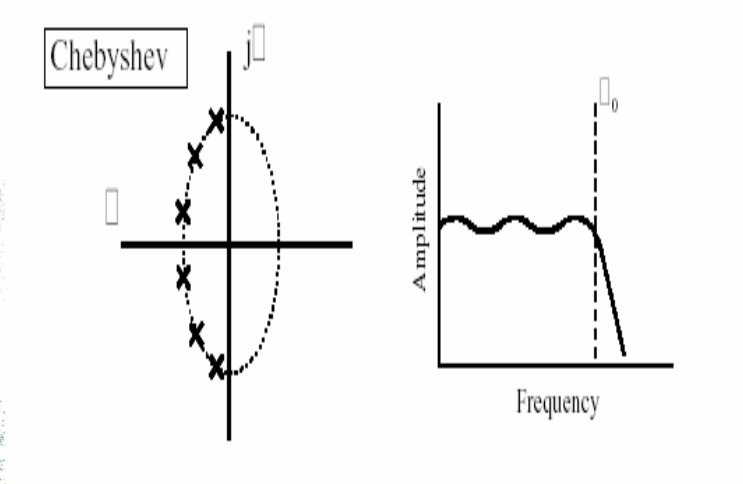
Butterworth filter
Band Pass

$$\overline{B(\omega)}B(\omega) = \frac{1}{1+\left(\frac{\omega^2}{\omega_c^2}\right)^{2n}}$$

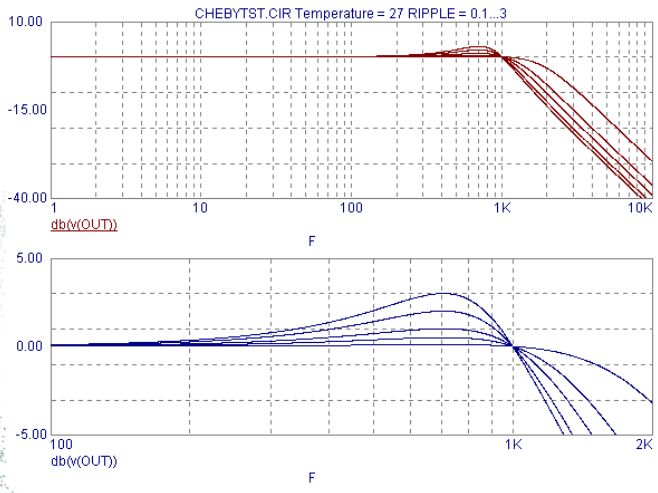




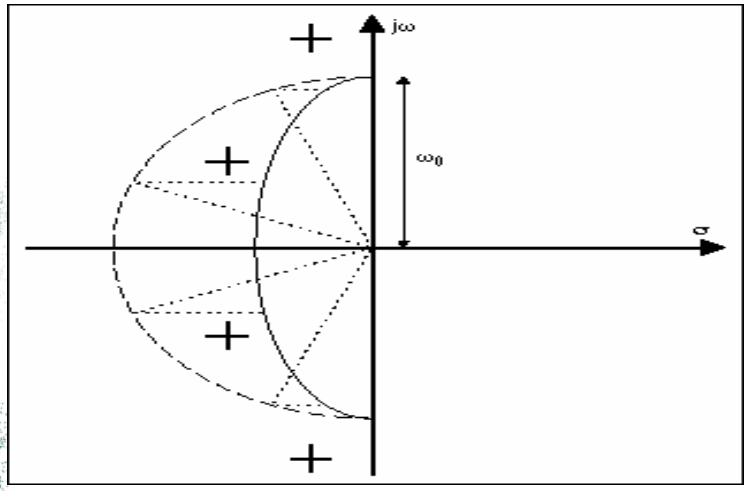
Chebyshev filter (3 poles)



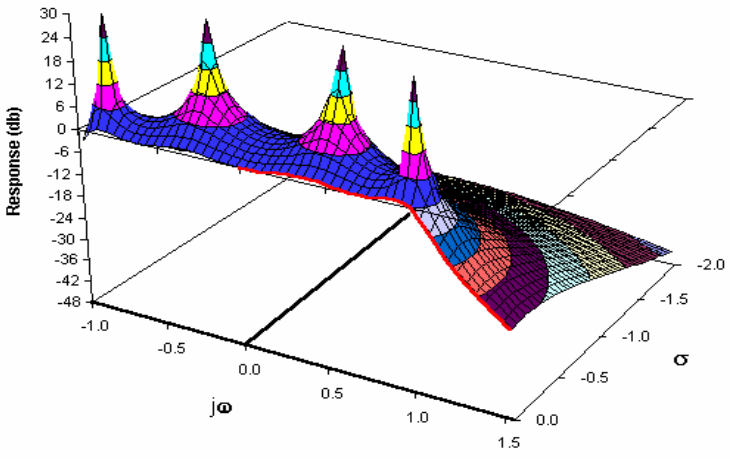
Chebyshev filter



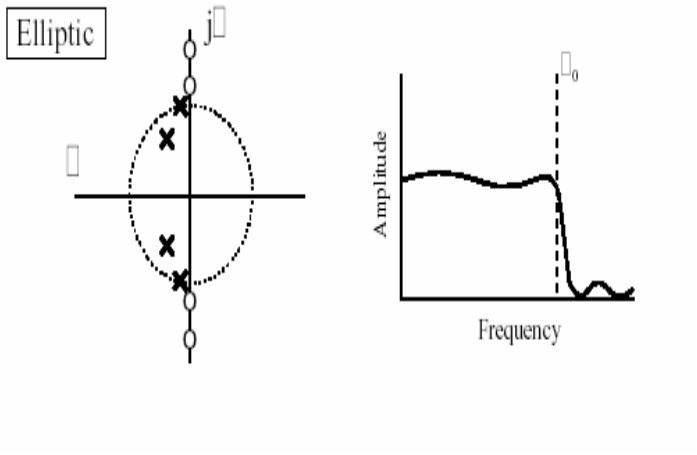
Chebyshev filter (4 poles)



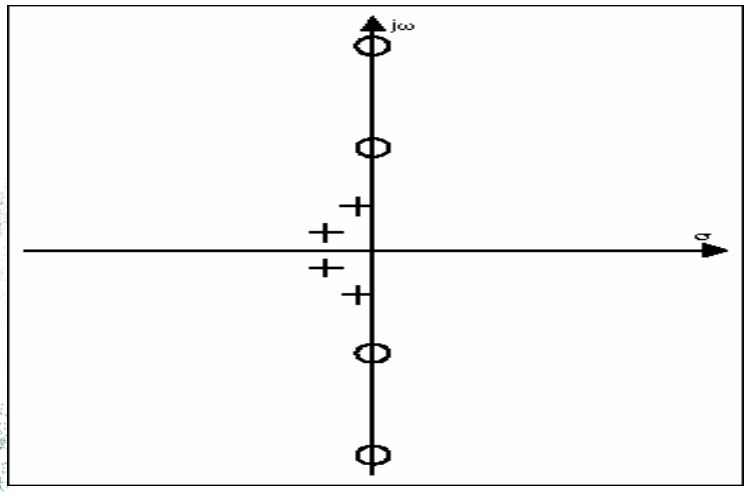
Chebyshev filter (4 poles)



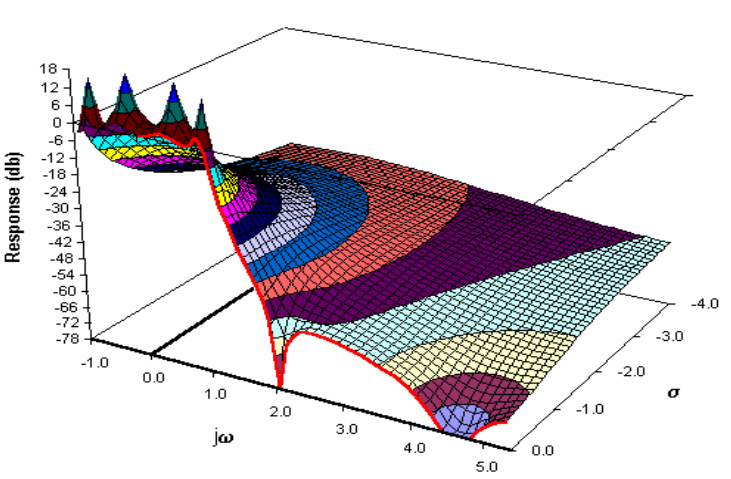
The elliptic filter



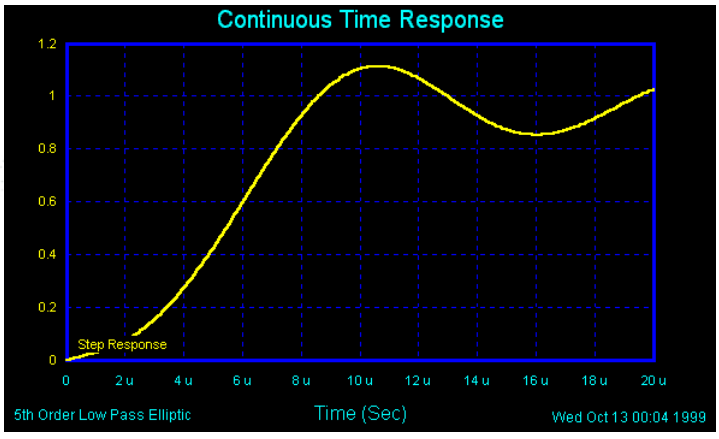
The elliptic filter



The elliptic filter



The elliptic filter



The Bessel Filter