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DIGITAL TERRAIN MODELLING

R WEIBEL AND M HELLER

Digital terrain models (DTMs) are a major constituent of geographical information processing. DTMs help to model, analyse and display phenomena related to topography or similar surfaces. This chapter is an attempt at a comprehensive review of relevant techniques and applications of DTMs. The individual elements of digital terrain modelling – techniques for the generation, manipulation, interpretation (analysis) and visualization of DTMs – are outlined. Application domains that use DTMs and their specific functional requirements are identified. Due to the complexity of the topic this chapter should be considered primarily as an annotated guide to the rich literature on digital terrain modelling.

INTRODUCTION

Digital terrain models (DTMs) have been used in geoscience applications since the 1950s (Miller and Laflamme 1958). Since then they have become a major constituent of geographical information processing. They provide a basis for a great number of applications in the earth and the engineering sciences. In GIS, DTMs provide an opportunity to model, analyse and display phenomena related to topography or other surfaces.

A DTM may be understood as a digital representation of a portion of the earth's surface. Since overhanging cliffs and faults are relatively rare in nature, topographic surfaces are most often represented as 'fields' (i.e. simply connected surface models, having unique z-values over x and y). In this sense a DTM is a '2.5-D' rather than a 3-D model. Extensions to model cliffs and faults are commonly provided by application-specific systems, where they may be crucially important, such as those used in modelling geological surfaces (see, for instance, McCullagh 1988; Raper and Kelk 1991 in this volume). Some authors argue that the term 'Digital Elevation Model' (DEM) should be used instead of 'Digital Terrain Model' when merely relief is represented, 'because the term "terrain" often implies attributes of a landscape other than the altitude of the landsurface' (Burrough 1986:39). Although this point is well taken, here the term 'digital terrain model' is intentionally used since it allows the possibility of including landscape attributes other than topography, as a means of improving the digital representation of a section of terrain. In a more general sense, a DTM may be used as a digital model of any single-valued surface (e.g. geological horizons and even air temperature or population density). Here, however, attention is focused on digital models of terrain since many aspects of the digital modelling of other surface phenomena are functionally related to terrain modelling.

As will be shown in the course of this chapter, the input data, data models and algorithms required by digital models of terrain or other surfaces are quite different from those used in representing planimetric (i.e. two-dimensional) data. The activity of modelling and processing digital terrain data may thus be regarded a system component of GIS that is functionally disparate from modelling 2-D data, yet needs to be closely linked to other processing functions of GIS (e.g., polygon, network and raster processing). Digital terrain modelling encompasses the following general tasks (cf. Weibel and Heller 1990):

- *DTM generation*: sampling of original terrain data, formation of relations among the diverse observations (model construction);
- *DTM manipulation*: modification and refinement of DTMs, derivation of intermediate models;
- *DTM interpretation*: DTM analysis, information extraction from DTMs;
- *DTM visualization*: graphical rendering of DTMs and derived information;
- *DTM application*: development of appropriate application models for specific disciplines. DTM application forms the context for digital terrain modelling: each particular application has its specific functional requirements relative to the other terrain modelling tasks.

Flexibility and adaptability to given problems are fundamental objectives of a digital terrain modelling system. Thus, as Fig. 19.1 shows, deriving products from a DTM should not be viewed as a one-way process, but rather as the result of various interrelated stages in modelling. For example, a DTM may be modified by model manipulation procedures. It might then be displayed by visualization procedures, or analysed through interpretation functions. Visualization and/or interpretation in turn may require or support further modification or adaptation of the original DTM. Thus, results of individual modelling steps may feed back into previously run procedures.

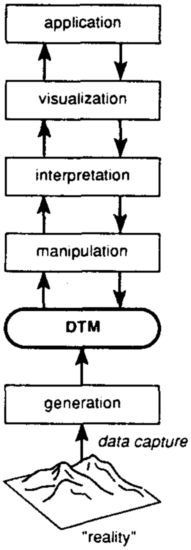


Fig. 19.1 The main tasks of a digital terrain modelling system.

In the course of this chapter the individual constituents of a comprehensive DTM system are gradually introduced. The text is structured according to the tasks of digital terrain modelling listed above. Given the restrictions in space, the discussion concentrates on a comprehensive overview of relevant techniques and applications. The functional interdependencies between the various processing tasks are identified and references are provided to further reading material and information about sources of current research. One of the objectives is that this text may be used as an annotated directory to the key literature on DTMs.

DTM GENERATION: TERRAIN DATA CAPTURE AND CONSTRUCTION OF DIGITAL TERRAIN MODELS

Data sources and data capture for digital terrain models

The choice of data sources and terrain data sampling techniques is critical for the quality of the resulting DTM. Data for a DTM should consist of observations about terrain elevations and, whenever possible, additional information about phenomena that significantly influence the shape of the terrain surface (i.e. structural features such as drainage channels, ridges and other surface discontinuities). There may be other criteria apart from DTM quality requirements, which will guide the selection of a particular sampling technique and scheme for any given application (e.g. efficiency, cost and technological maturity).

At present, most DTM data are derived from three alternative sources: ground surveys, photogrammetric data capture (in manual, semi-automated, or automatic mode), or from digitized cartographic data sources. Other methods occasionally used include radar or laser altimetry, and sonar (for subaquatic terrain). Data for geological models are obtained from either borehole records or seismic surveys.

As a further alternative, models of artificial terrain may be created through digital simulation, as noted by McLaren and Kennie (1989). Fournier, Fussel and Carpenter (1982) have proposed a method which is frequently used to generate quasi-realistic images of terrain. However, this technique is based solely on modelling stochastic processes. Other authors (Griffin 1987; Clarke 1988; Kelley, Malin and Nielson 1988; Musgrave, Kolb and Mace 1989; Szelinski and Terzopoulos 1989) have also tried to incorporate geomorphological simulation into their models.

Ground surveys

Survey data may be input directly into computer systems through data recorders which may be coupled to field instruments. Since ground survey data tend to be very accurate, and surveyors tend to adapt the survey to the character of the terrain (i.e. they measure significant terrain points), the accuracy of the resulting DTM is very high. However, as this particular data collection technique is relatively time consuming, its use is limited to small areas. Thus, ground surveys are commonly applied to specific projects (e.g. site planning of small areas) or used to complement photogrammetric data capture (e.g. to provide data for wooded areas).

Photogrammetric data capture

Photogrammetric data capture is based on the stereoscopic interpretation of aerial photographs or satellite imagery (e.g. from the French SPOT satellites), using suitable photogrammetric equipment (i.e. manual or analytical stereoplotters). It is possible to distinguish a number of different photogrammetric sampling techniques: regular sampling patterns, progressive sampling, selective sampling, composite sampling, and digital stereo image correlation for automatic DTM extraction. Each of these methods attempts to minimize the data collection effort (i.e. the number of elevation samples to be taken), while at the same time maximizing the accuracy of the resulting DTM. Depending on the sampling method and imagery that are used, the resulting DTM accuracy will be medium to high. Since remote sensing is used instead of field work, large areas may be handled in a relatively short amount of time. Photogrammetric data capture is used in large engineering projects (e.g. dams, open-cast mines and roads) as well as nationwide data collection.

The simplest photogrammetric sampling technique is that of regular sampling patterns. Regular patterns may be arranged as profiles or grids (Fig. 19.2(a) and (b)). Since a fixed sampling distance is used, it is important to consider the determination of the optimal sampling interval. Blais, Chapman and Lam (1986) discuss several strategies for this problem, including techniques based on spectral analysis, linear interpolation and variogram estimation. The advantage of regular sampling patterns is that they may be applied in a semi-automated or even automatic mode (if correlators are used). However, due to the fixed sampling distance, the usefulness of this technique is restricted to fairly low and homogeneous terrain. An excessive number of points tends to be sampled in regions of low relief, whereas too few points are captured in rugged terrain.

In 'progressive sampling', developed by Makarovic (1973), the density of sample points is adapted to the complexity of the terrain surface. The sampling process is initiated by measuring a

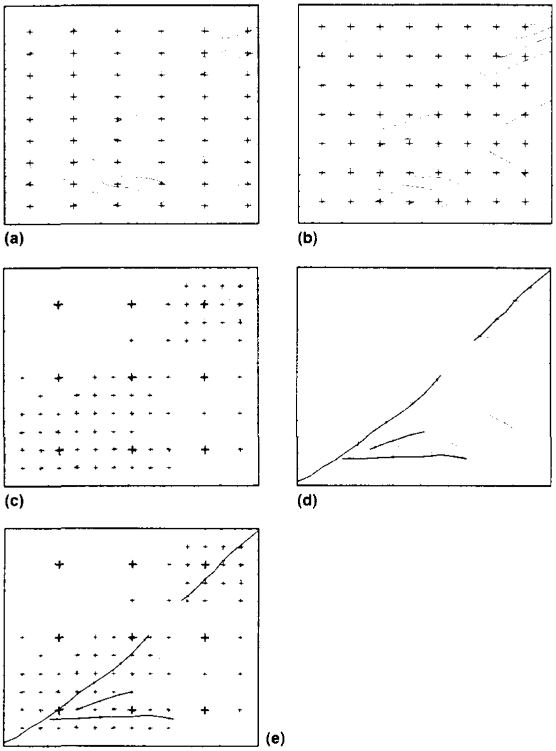


Fig. 19.2 Photogrammetric sampling techniques: (a) regular sampling pattern (profiles); (b) regular pattern (grid); (c) progressive sampling; (d) selective sampling; (e) composite sampling.

low density grid. The accuracy of the sampled data is then analysed. Wherever necessary, the sampling grid is recursively densified until the required accuracy level is reached. This results in a hierarchical sampling pattern (Fig. 19.2(c)). The method of progressive sampling may also be automated. Due to the adaptivity of the progressive sampling process, fewer points are needed to produce DTMs of higher accuracy than for regular sampling patterns. However, since a regular densification pattern is used, it still requires too many points to represent terrain breaks accurately.

In regions of sharp terrain discontinuities, it is sometimes necessary to capture topographic breaks selectively (Fig. 19.2(d)). This sampling method is termed 'selective sampling' (Makarovic 1984). If selective sampling of structural features is combined with progressive sampling, the technique is called 'composite sampling' (Makarovic 1977, 1979), in which selective sampling is used to capture abrupt topographic changes while progressive sampling yields the data for the rest of the terrain (Fig. 19.2(e)). Thus, the recursive refinement of progressive sampling can be kept to a minimum and terrain discontinuities are represented accurately. Composite sampling yields terrain data of high quality. However, since the selective sampling of structural features requires human intervention, this method may only be partially automated.

There has been an increased tendency during the past decade towards the fully automated extraction of terrain models through correlation of digital stereo images. This correlation can be achieved either by using correlation devices, or 'off-line', that is fully computationally without involvement of any photogrammetric equipment (Konecny and Pape 1981; Barnard and Fischler 1982; Day and Muller 1988; Lemmens 1988). In digital stereo correlation, corresponding picture elements or features are matched through cross correlation in order to obtain parallaxes and derive elevations. Although this technique is fast and seems suitable for large data collection projects, data quality is generally not very high since errors may be easily introduced (e.g. through pattern mismatches or through noisy digital data). Consequently, procedures for automated error detection and correction have to be devised to increase the data quality.

Cartographic data sources

It is also possible to derive DTMs from cartographic documents, such as contour maps and profiles. These analogue data may be digitized through manual digitization, semi-automated line-following, or by means of automatic raster scanning and vectorization. Due to the relatively high costs of direct methods for terrain data capture (i.e. surveying and photogrammetry) and the large volume of existing paper maps, this indirect method is predominant for large data collection projects. This is particularly true for national or military mapping agencies. Figure 19.3 shows a sample of scan-digitized contour lines generated for a project of this type. Despite their widespread use as a basis for DTMs, contour data present some problems. Contours are mainly a form of terrain visualization and are not particularly useful as a scheme for numerical surface representation. An excessive number of points is sampled along contours (oversampling), and no data across contours (undersampling). Furthermore, errors may be introduced (in drawing, line generalization, reproduction, etc.) and a lot of the original information is lost in the mapmaking process. Consequently, contour data yield DTMs of only limited accuracy. However, since large area coverage is achieved relatively cost effectively, digitized cartographic documents provide a compromise method of obtaining DTMs for use at medium or small scales.

Model construction

The process of terrain data capture generates a set of relatively unordered data elements (the 'original observations'). In order to construct a comprehensive DTM it is necessary to establish the topological relations between the data elements, as well as an interpolation model to approximate the surface behaviour.

Data structures for digital terrain models

The original data must be structured to enable handling by subsequent terrain modelling operations. A variety of data structures for DTMs has been in use over time (Peucker 1978; Mark 1979). Today, however, the overwhelming majority of DTMs conform to one or other of two data

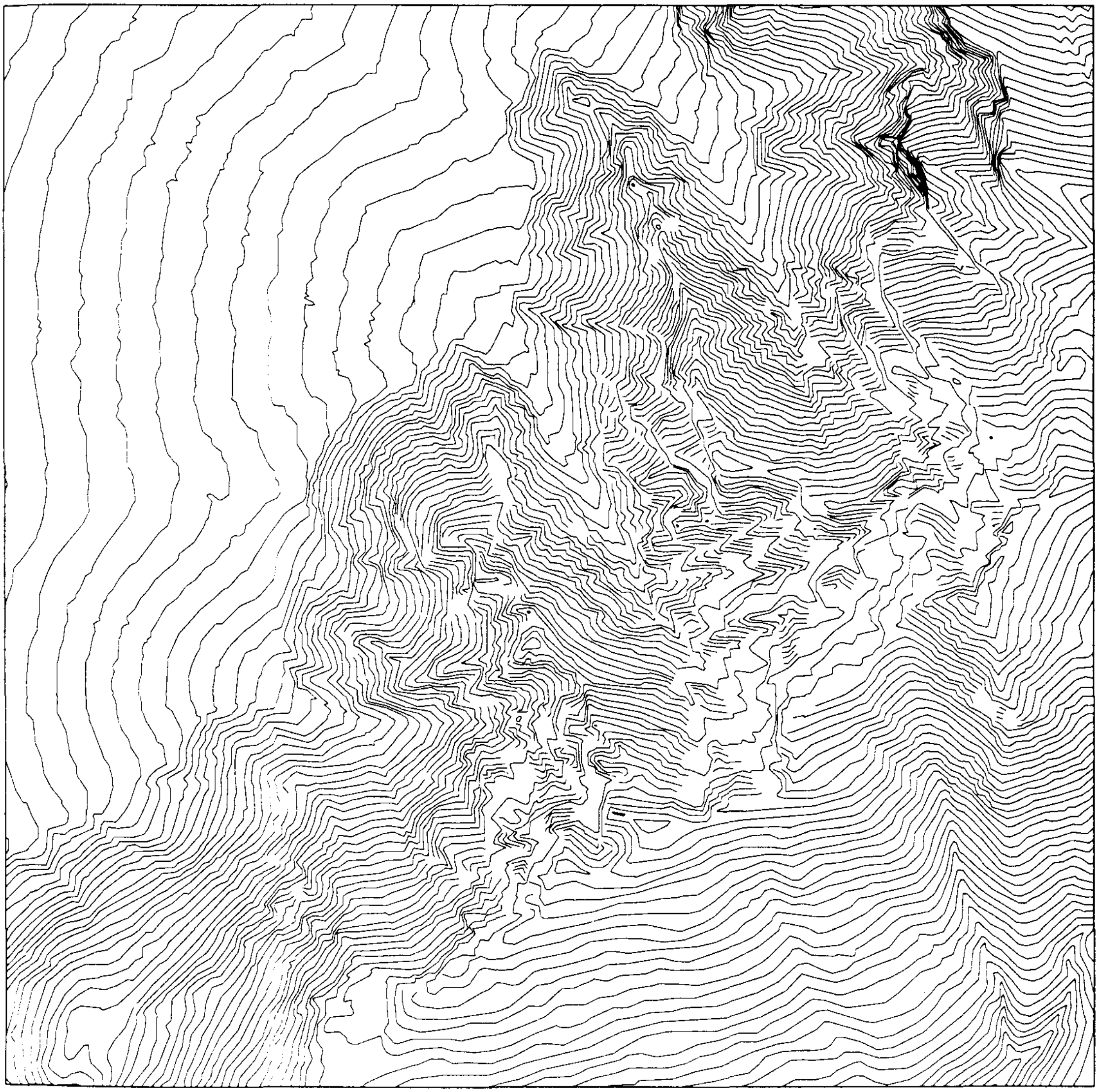


Fig. 19.3 Sample of scan-digitized and vectorized contour lines from the National Map of Switzerland at 1 : 25 000 scale. (Courtesy of Swiss Federal Office of Topography, 22.12.89)

structures: rectangular grid (or elevation matrix), or TIN (Triangulated Irregular Network; Peucker *et al.* 1978).

Grids present a matrix structure that records topological relations between data points implicitly (Fig. 19.4(a)). Since this data structure reflects the storage structure of digital computers (i.e. a grid can be stored as a two-dimensional array of elevations), the handling of elevation matrices is simple, and grid-based terrain modelling algorithms thus tend to be relatively straightforward. On the other hand, the point density of regular grids cannot be adapted to the complexity of the relief. Thus, an excessive number of data points is needed to represent the terrain to a required level of accuracy. Also, rectangular grids cannot describe structural features as topographic features; extensions to the basic model have to be added for this purpose (cf. Köstli and Sigle 1986; Ebner, Reinhardt and Hössler 1988).

TIN structures, on the other hand, are based on triangular elements, with vertices at the sample

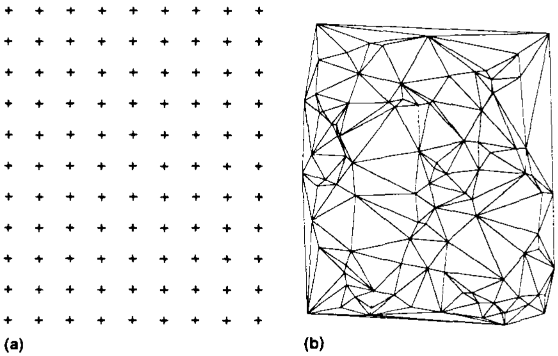


Fig. 19.4 The two most commonly used data structures for DTMs: (a) rectangular grid (or elevation matrix, gridded DTM); (b) Triangulated Irregular Network (TIN).

points (Fig. 19.4(b)). Structural features can easily be incorporated into the data structure. Consequently, TINs are able to reflect adequately the variable density of data points and the roughness of terrain. Fewer points are needed for a DTM of a certain accuracy. However, topological relations have to be computed or recorded explicitly. Thus, TINs become more complex and also more difficult to handle. Not all grid-based algorithms today have an efficient counterpart in TIN structures.

Both Peucker (1978) and Mark (1979) discuss additional advantages and disadvantages of rectangular grids, TINs and other DTM data structures so there is no need to dwell further on this subject here. It is more important to note that no data structure is clearly superior for all tasks of digital terrain modelling. Thus, the ability to switch between different data structures, as well as modify and refine the DTM become prime objectives of any flexible DTM system. To facilitate these processes, the system should be based on a data model that is general enough to accommodate a variety of uses: the original observations act as the basis for this model, and specific representations (such as TINs, grids, etc.) are derived from these and act as interfaces to DTM procedures (Weibel *et al.* 1987; Weibel and Heller 1990). These intermediate models only hold that particular subset of the basic DTM which is necessary for solving a given problem. The framework for such a system is described in more detail in Weibel and Heller (1990).

Interpolation

In digital terrain modelling, interpolation serves the purpose of estimating elevations in regions where no data exist. Interpolation is mainly used for the following operations:

- computation of elevations (z) at single point locations;
- computation of elevations (z) of a rectangular grid from original sampling points;
- computation of locations (x,y) of points along contours (in contour interpolation);
- densification/coarsening of rectangular grids (so-called resampling).

Abundant literature exists on methods for interpolation of DTMs. A number of articles may be used as an entry point to the exploration of different interpolation models (e.g. Schut 1976; Schumaker 1976; Lam 1983; Heller 1986; McCullagh 1988). Lam (1983) groups point interpolation into exact and approximate methods. The former preserve the values at the data points, while the latter smooth out the data. Another popular way to classify interpolation models is by the range of influence of the data points involved. Global methods, in which all sample points are used for interpolation may be distinguished from local, piecewise methods, in which only data points nearby are considered. Because topographic surfaces are non-stationary and non-periodic the use of overly distant points may deform the interpolated surface. For DTMs with sample points of sufficient quality and density, a local and exact interpolation method on surface patches is widely considered satisfactory.

There is insufficient space here to discuss individual interpolation models, rather it seems appropriate to refer to the rich literature and just briefly mention the characteristics and peculiarities of DTM interpolation from topographic samples:

- There is no 'best' interpolation algorithm that is clearly superior to all others and appropriate for all applications (e.g. see Lam 1983);
- The quality of the resulting DTM is determined by the distribution and accuracy of the original data points (i.e. the sampling process), and the adequacy of the underlying interpolation model

(a hypothesis about the behaviour of the terrain surface);

- The most important criteria for selecting a DTM interpolation method are the degree to which (1) structural features can be taken into account, and (2) the interpolation function can be adapted to the varying terrain character;
- Suitable interpolation algorithms must adapt to the character of data elements (type, accuracy, importance, etc.) as well as the context (i.e. the distribution of data elements). Satisfactory solutions exist for the interpolation of relatively well-selected and dense topographic samples (e.g. photogrammetric data). There are, however, a number of critical cases that still pose problems for current procedures (see Figure 19.5);
- Other criteria that may influence the selection of a particular method are the degree of accuracy desired and the computational effort involved.

The above considerations only partially apply to the interpolation of non-terrain surfaces. However, some of the review articles mentioned earlier (e.g. Lam 1983; Schumaker 1976) also discuss the interpolation of these surfaces.

Triangulation

Some of the more widely used local interpolation procedures are based on triangulation: interpolation is achieved by locally fitting polynomials to triangles (the simplest case being linear interpolation within triangles). Furthermore, triangulation is used to construct TIN DTMs, as discussed earlier. Triangulation thus serves two purposes in terrain modelling: as a basis for TIN data structures, and as a basis for interpolation (e.g. of gridded DTMs or contours). However, TIN-based interpolation (in conjunction with the interpolation of gridded DTMs) is probably more common than TINs used as DTM data structures. TIN-based interpolation is essentially a two-step procedure: in the first step a TIN is constructed; it is then used for interpolation in the second step.

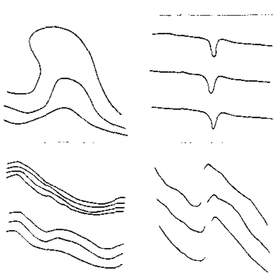


Fig. 19.5 Some critical cases for interpolation. For instance, interpolation procedures have to be able to interpret the fine structure of neighbouring contour lines and extend it across the area between lines. Note that such situations are not uncommon in real data (cf. Fig. 19.3).

Of the many possibilities available to triangulate a set of points, the so-called 'Delaunay triangulation' has received particular attention. An example of such a triangulation is shown in Fig. 19.6(a). A triangulation of a set of points is a Delaunay triangulation if, and only if, the circumcircle of any of its triangles does not contain any other point in its interior. The Delaunay triangulation is bounded by the convex hull. The dual of the Delaunay triangulation is called a Voronoi diagram (or Thiessen polygons, or Dirichlet tessellation). Nodes of a Voronoi polygon are coincident with the circumcentres of Delaunay triangles (Fig. 19.6(b)). The perpendicular bisectors of Delaunay edges form the edges of Voronoi polygons. Since they are dual, the Delaunay triangulation may be constructed from its Voronoi diagram and vice versa. A more mathematical and profound treatment of Delaunay triangulation may be found in Preparata and Shamos (1985), or Edelsbrunner (1987). It is interesting to note that because of their specific properties, Delaunay triangulation and the Voronoi diagram have found widespread use in many fields other than surface modelling (e.g. computational geometry, physics, meteorology, economic geography, etc.). A number of efficient algorithms for constructing a Delaunay triangulation (or Voronoi diagrams) have been described (e.g. Guibas and Stolfi 1985; Preparata and Shamos 1985; Edelsbrunner 1987; Heller 1990). Heller (1986) provides further annotated references. Figure 19.7 shows a triangulated set of terrain spot heights.

It is important to observe that any product which is derived from a TIN – whether it is a perspective display, a set of slope values, or an interpolated grid DTM – will be heavily dependent upon the quality of the TIN. Since a TIN is

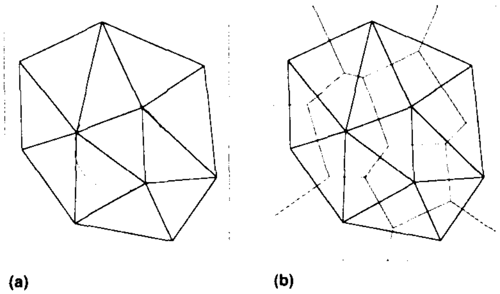


Fig. 19.6 The concept of Delaunay triangulation: (a) Delaunay triangulation with circumcircle of one of the triangles; (b) Delaunay triangulation and its dual, the Voronoi diagram (or Dirichlet tessellation or Thiessen polygons).

primarily a topological structure, quality also relates to the logical consistency of the TIN, that is whether or not the formation of the triangles complies with the geomorphological facts of the terrain surface being modelled. Obviously, several of the critical cases illustrated in Figs. 19.3 and 19.5 cannot be adequately handled by pure Delaunay triangulation (Fig. 19.8(a), and 19.8(c)). Thus, the Delaunay criterion has to be relaxed. The triangulation must be constrained so that segments

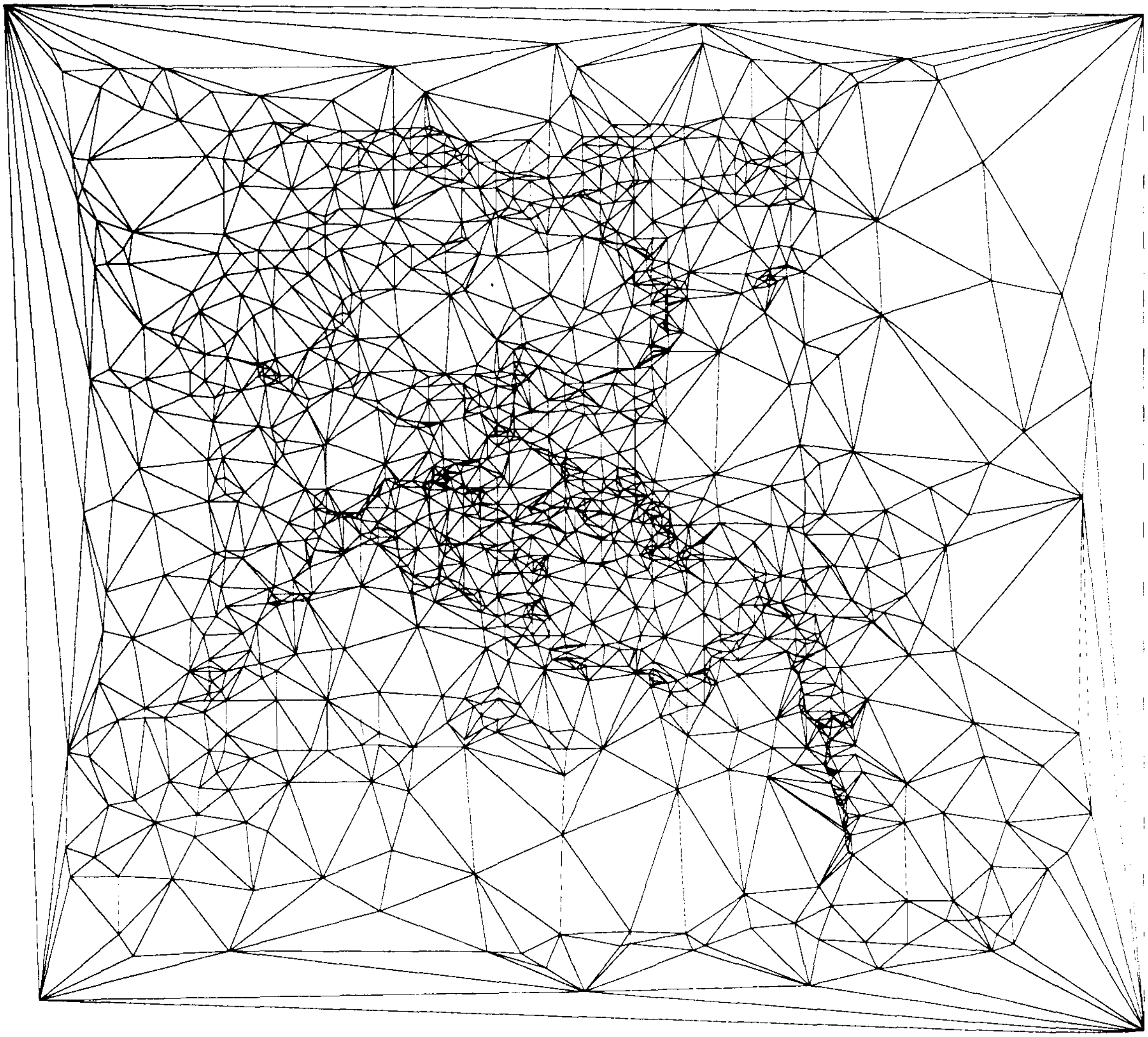


Fig. 19.7 Delaunay triangulation of a set of terrain spot heights.

of discontinuities or other features of spatial coherence form edges of triangles (Fig. 19.8(b)). The triangulation must also be adjusted for interpolation from contour data (Fig. 19.8(d)). Although several computer programs exist today that construct constrained triangulations, only a few authors have actually reported on their solution (e.g. McCullagh 1982; Christensen 1987; McCullagh 1988; Heller 1990).

Once the TIN has been constructed in the triangulation process, it can either be used directly as a TIN DTM, or else as a basis for interpolation. In TIN-based interpolation, the (interpolated) *z*-value of a point depends on the heights of the nodes of the triangle that contains it. The *z*-value is interpolated by substituting the *x*,*y* values of the point to a polynomial function that is fitted to the relevant triangle. If linear interpolation is used, the *z*-value can be computed directly from the heights of the nodes of the triangle. If a higher order function is applied (e.g. Birkhoff and Mansfield 1974; Akima 1978), coefficients of that function are estimated based on the heights of the triangle nodes, as well as their first and second order derivatives. The derivatives at a node depend on the heights of all its neighbours (i.e. nodes with which it shares a common edge). If discontinuities are included in the triangulation, no heights of nodes across a discontinuity are included in the computation of the derivatives. McCullagh (1982, 1988) demonstrates his approach to TIN-based interpolation in detail.

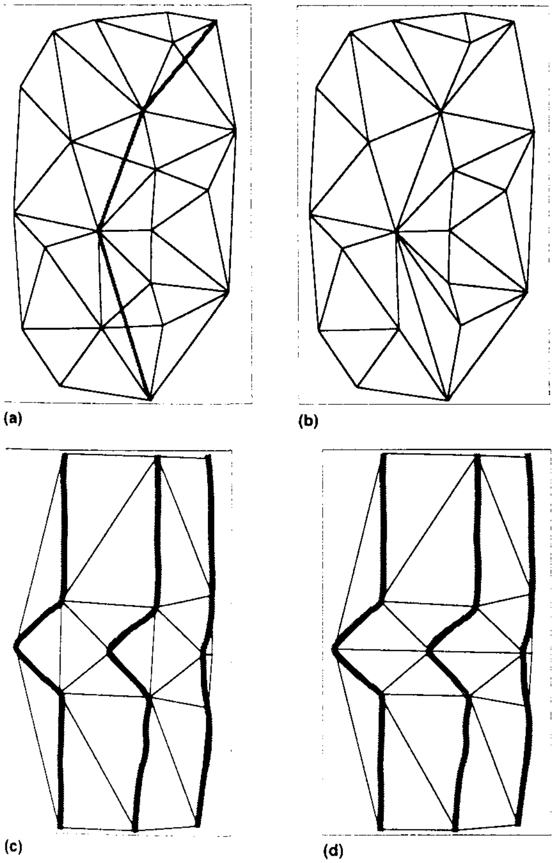


Fig. 19.8 Pure Delaunay triangulation may conflict with linear constraints: (a) Delaunay edges cross linear discontinuity; (b) constrained triangulation; triangle edges are coincident with linear constraints; (c) triangles have been formed of points on the same contour line; (d) adjusted triangulation.

One of the main advantages of TIN-based interpolation is that it is very efficient. The first phase is independent of the second one. Once the TIN has been constructed, gridded DTMs or contours may be efficiently computed since the TIN provides a convenient means of locating the relevant nodes for interpolation. Further advantages of TIN-based interpolation are that it is a local method which allows discontinuities to be incorporated easily, and that it may be used to transform any TIN into a gridded DTM (i.e. data structure conversion).

Because of its local scheme and efficiency, TIN-based interpolation is one of the few methods that may be considered practicable even for very large point sets. To date, developments of TIN-based interpolation algorithms have focused on memory-resident methods. To manage very large data sets, disk-based algorithms should be used which divide the plane into regions (tiling), triangulate each region separately, and eventually patch the triangulated sub-areas together (joining).

A special case of interpolation: interpolation from contour data

As mentioned earlier, digitized contours are not a particularly useful digital representation of terrain, yet they remain a popular data source for DTMs. Furthermore, many of the special cases of Fig. 19.5 may be found in real contour data such as those displayed in Fig. 19.3. Interpolation from contours thus seems to be especially complex and certainly deserves special attention. Several contour-specific interpolation algorithms have been developed. Most algorithms for interpolating regular grid DTMs (e.g. Clarke, Grün and Loon 1982; Oswald and Raetzsch 1984) use linear or cubic interpolation along straight lines (either along predefined directions or in the direction of steepest slope), 'in an attempt to extend across the areas between the contour lines at least some of the fine structure contained in the data contours' (Hutchinson 1988:126). Clarke, Grün and Loon (1982) present a thorough review and evaluation of this class of contour-to-grid algorithms. However, these methods are still not capable of resolving some of the critical cases illustrated in Fig. 19.5.

A more sophisticated procedure has been presented by Hutchinson (1988, 1989). His method includes a drainage enforcement algorithm, which automatically removes spurious sinks or pits in the fitted elevation grid, and an algorithm which automatically calculates ridge and stream lines from points of maximum local curvature on contour lines. These two techniques allow a more reliable interpolation of the fine structure in contours across the area between data contour lines. A partly similar approach that first computes aspect vectors from the contour data to enhance the interpolation model has been reported by Inaba, Aumann and Ebner (1988).

Apart from enhancing contour-to-grid interpolation methods, there has also been recent and growing interest in improving triangulation-based interpolation. The method of Christensen (1987), using the medial axis of contour polygons for triangulation, shows considerable improvement over Delaunay triangulation. However, it results in an overkill, as normally only few cases would actually require departure from the Delaunay criterion. Consequently, this procedure is not very efficient. Other approaches that attempt to recognize critical cases in contour data and locally relax the Delaunay criterion, are currently still under research.

DTM MANIPULATION: MODIFICATION AND REFINEMENT OF DTMS

Along with DTM generation procedures, the DTM *manipulation processes* are of fundamental importance for the performance and flexibility of a DTM system. They are needed for the modification and refinement of existing models. DTM manipulation consists of processes for DTM editing, filtering and merging, and for the conversion between different data structures.

DTM editing

DTM *editing* involves updating and error correction. An editor is required for interactive, selective modification of the properties of individual elements of a DTM. Edit operations for DTM elements should include: query, delete, add, move, change height, change attribute, etc. For gridded DTMs, however, editing is essentially restricted to modifying elevations at grid points. If TINs are edited, algorithms are required for local adjustment of the network topology after points have been inserted or deleted (Preparata and Shamos 1985; Heller 1990). Furthermore, it is helpful if an effective user interface is used. Interaction by direct manipulation supported by visual feedback (e.g. Apple 1987) will greatly simplify the editing task.

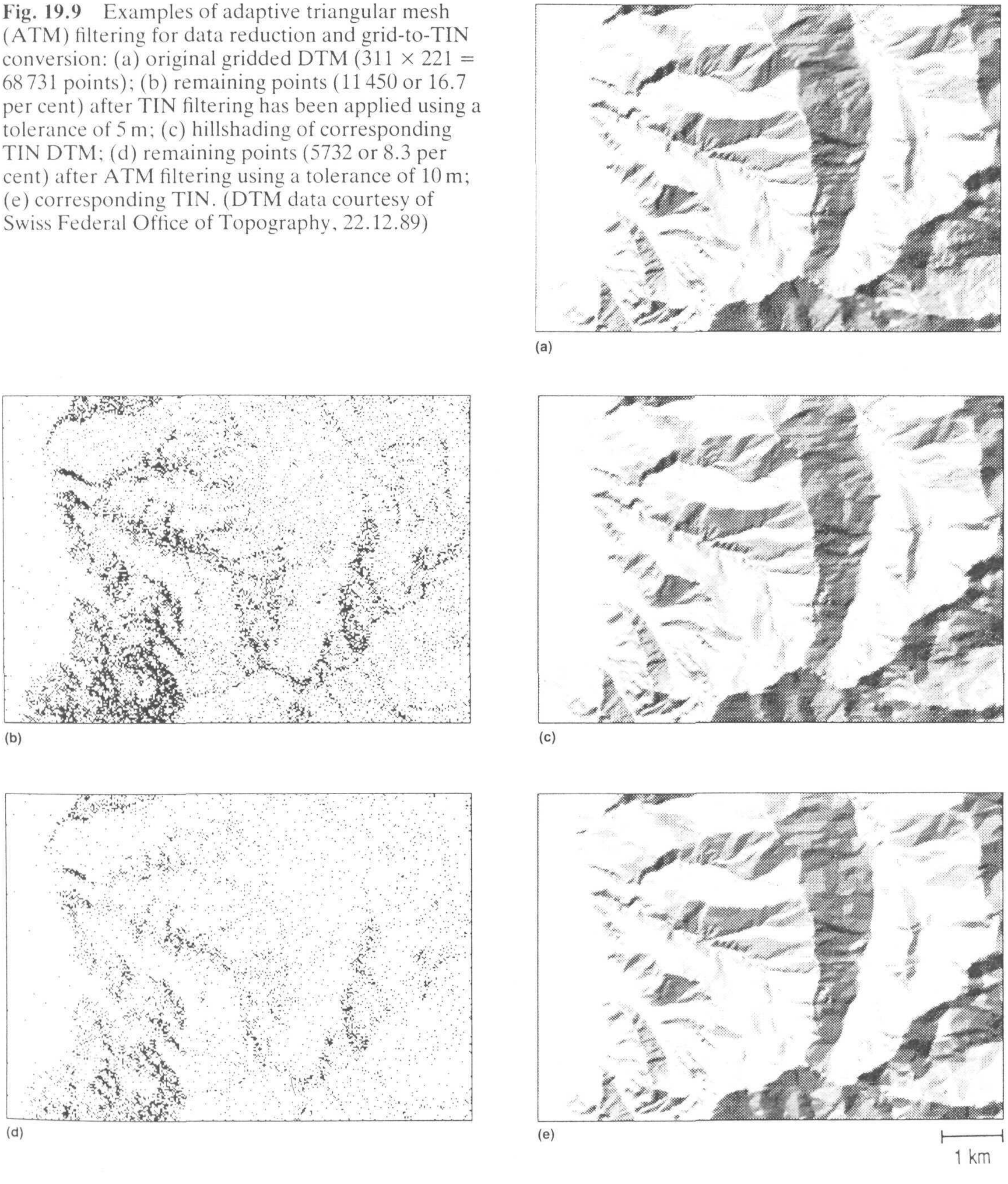
DTM filtering

DTM filtering may serve two purposes: smoothing or enhancement of DTMs, as well as data reduction.

Smoothing and enhancement filters for DTMs are equivalent to lowpass and highpass filters as they are known in other fields, such as image processing (e.g. see Gonzalez and Wintz 1987). They may be applied both in the spatial domain (by moving average operations) or in the frequency domain (by convolution of Fourier transforms). They are best applied to gridded terrain models. The effect of smoothing (i.e. applying lowpass filters) is to remove details, and make the DTM surface smoother. Enhancement (i.e. highpass filtering) has the opposite effect: surface discontinuities are emphasized, while smooth shapes are suppressed. Smoothing filters have been used to eliminate blunders (e.g. in photogrammetric data); they may also be used to some extent for cartographic generalization of DTMs (Zoraster, Davis and Hugus 1984; Weibel 1989; Muller 1991 in this volume). Enhancing (or highpass) filters have only rarely been applied to DTMs.

DTM filtering procedures are also used to

Fig. 19.9 Examples of adaptive triangular mesh (ATM) filtering for data reduction and grid-to-TIN conversion: (a) original gridded DTM (311 × 221 = 68 731 points); (b) remaining points (11 450 or 16.7 per cent) after TIN filtering has been applied using a tolerance of 5 m; (c) hillshading of corresponding TIN DTM; (d) remaining points (5732 or 8.3 per cent) after ATM filtering using a tolerance of 10 m; (e) corresponding TIN. (DTM data courtesy of Swiss Federal Office of Topography, 22.12.89)



reduce the data volume of DTMs. A data reduction process of this kind may be desirable to eliminate redundant data points (e.g., within the digitizing tolerance), to save storage space and processing time, or to reduce a DTM's resolution (e.g., for comparison with other models). It may also be used as a pre-processing step in cartographic generalization of DTMs (Weibel 1989), or to convert a gridded DTM into a TIN (see below).

This problem has been addressed by Gottschalk (1972). As his method uses a global interpolation scheme, it is only practical for small data sets. An efficient approach – called adaptive triangular mesh (ATM) filtering – has been developed by Heller (1990). A given triangulation is fitted to a specified local height tolerance. The procedure is based on an algorithm that repeatedly adds the most significant point to the triangulation, until no additional points are needed to describe the surface within the specified tolerance. The importance of a point is given by the vertical distance (i.e. elevation difference) between the point and the retriangulated model without that point. The procedure builds a distance queue, adds the point with the largest distance, retriangulates its neighbourhood, updates the queue, and continues to insert points until the largest distance is smaller than the specified tolerance.

This algorithm may be used to reduce data volumes of both TINs and gridded DTMs. A grid may be considered as a special case of a TIN, with nodes arranged in a rectangular grid. Thus, grids may be triangulated and filtered in the same way other TINs would be processed. However, as some points will be deleted through the filtering process, grids will be transformed into TINs. An example of ATM filtering is shown in Fig. 19.9: starting from a gridded DTM (68 731 points), a TIN DTM with a tolerance of 5 m (with 11 450 remaining points or 16.7 per cent), and a TIN DTM with a tolerance of 10 m (5732 points or 8.3 per cent) were obtained.

DTM joining and merging

DTMs may be combined either by joining adjacent models or by merging overlapping models.

For gridded DTMs, joining is only straightforward if the grids correspond in grid resolution, orientation, etc. Otherwise, a resampling process (for coarsening, densification, and/or reorientation) has to be used to establish continuity. The joining of TINs requires algorithms for connection and readjustment of the TINs along their borders (zipping) to patch the models together (Guibas and Stolfi 1985; Preparata and Shamos 1985; Heller 1990).

Merging DTMs poses problems at two levels. The first task consists of inserting all elements of one model into the other model (e.g. by incremental triangulation versus full re-triangulation). The second problem domain involves combining data sets with conflicting attributes (elevation and gradients) and varying degrees of accuracy. It is complex and represents a fundamental methodical problem.

Data structure conversion

The task of converting a DTM of a certain representation (e.g. TIN) into another structure (e.g. grid) can be handled by a combination of DTM generation and manipulation procedures.

As has been shown earlier, grid-to-TIN conversion may be handled by the same procedure used for data reduction of TINs (i.e. ATM filtering). In order to be able to exploit the benefits of a TIN data structure (i.e. adapt the point density to the terrain complexity to save storage space), insignificant points must be discarded in the conversion process. As Fig. 19.9 shows, a grid can be successfully converted to a TIN of substantially reduced data volume using ATM filtering. There are also strong indications that this algorithm is more suitable for handling grid-to-TIN conversion than other approaches (e.g. Chen and Guevara 1987).

Other conversions are essentially equivalent to interpolation processes discussed in the preceding section. TIN-to-grid conversion is equivalent to TIN-based interpolation of a gridded DTM. Contour-to-grid and contour-to-TIN conversion are special variants of interpolation tasks. Grid-to-contour and TIN-to-contour processes are solved by contour interpolation (covered in the section on visualization below).

Miscellaneous DTM manipulation functions

A number of further manipulation operations may be included in a DTM system, such as densification or coarsening of gridded DTMs (i.e. grid resampling by interpolation), or interpolation of z-values for 2-D features (e.g. drainage features and roads). These functions can be satisfied by a combination of basic functions of DTM generation and manipulation as discussed above.

DTM INTERPRETATION: AUTOMATED TERRAIN ANALYSIS TO SUPPORT GIS MODELLING

Within a GIS, digital terrain models are most valuable as a basis for the extraction of terrain-related attributes and features. Information may be extracted in two ways: by visual analysis of graphic representations (i.e. through visualization) or by

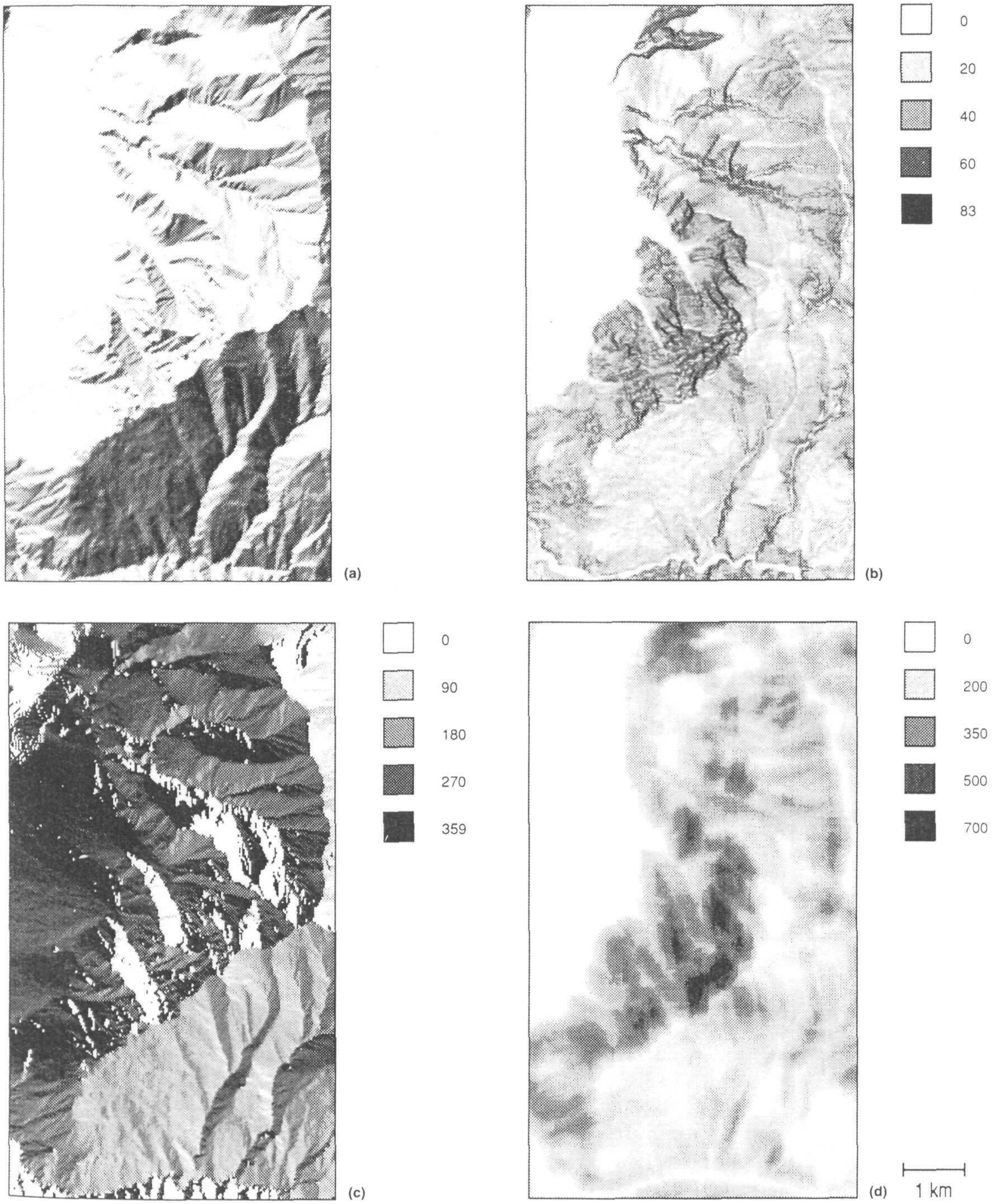


Fig. 19.10 Derivation of geomorphometric parameters from DTMs: (a) grid DTM (220 × 390 points); (b) corresponding map of gradient (in degrees); (c) map of aspect (in degrees); (d) map of local relief (range of altitude) within a 13 × 13 moving window. (DTM data courtesy of Swiss Federal Office of Topography, 22.12.89)

quantitative analysis of digital terrain data (i.e. through interpretation). Interpretation procedures, along with visualization functions, thus represent an important objective of GIS-related terrain modelling. The results of interpretation can be used as input to environmental impact studies, soil erosion potential models, hydrological runoff simulations, and many more applications.

Interpretation for geomorphometric analysis

A first objective of DTM interpretation is the derivation of geomorphometric parameters. According to Evans (1972:18), geomorphometric analysis may take two forms: general geomorphometry, 'the measurement and analysis of those characteristics of landform which are applicable to any continuous rough surface'; and specific geomorphometry, 'the measurement and analysis of specific landforms such as cirques, drumlins and stream channels, which can be separated from adjacent parts of the land surface according to clear criteria of delimitation'.

General geomorphometry

The most frequent use of general geomorphometry is the derivation of slope values from DTMs. Slope is defined by a plane tangent to the DTM surface at any given point, and comprises two components: gradient (maximum rate of change in altitude), and aspect (the compass direction of this maximum). This terminology has been used by Evans (1980); other terms frequently used include 'slope' to mean 'gradient' as just defined, and 'exposure' for 'aspect'. Examples of maps of gradient and aspect are shown in Figs. 19.10(b) and 19.10(c), respectively. Apart from being displayed as slope maps, gradient and aspect are often used as numerical input to GIS models, for example in trafficability studies or soil erosion modelling (e.g. Roo and Hazelhoff 1988). Besides gradient and aspect (i.e. the first derivatives of the altitude surface), the second derivative (or rate of change of slope) – convexity (or curvature) – is often used for geomorphological analysis. Convexity also has two components: profile convexity, i.e. the rate of change of gradient; and plan convexity, the convexity of contours (Evans 1980).

Although both slope and convexity are defined at a point, they are commonly assigned to DTM facets (i.e. triangles of TINs or rectangular cells of gridded DTMs) in slope analysis. Evans (1979) gives equations for the computation of gradient and aspect, and profile and plane convexity for points of a gridded DTM – by locally fitting a quadratic surface to a 3 × 3 submatrix. In an alternative approach, gradient and aspect may be determined using vector analysis. The z-component of the unit vector surface normal (in a given point) equals the cosine of gradient. Likewise, aspect is determined from the x- and y-components of the surface normal. The main task is to compute the surface normals of individual facets of the DTM. For triangles of a TIN, the surface normals are a by-product of the interpolation method (e.g. the cross-product of two vectors of the triangle in the linear case). For gridded DTMs, rectangular facets are formed from four adjacent grid points. Since rectangles are rarely plane, vectors have to be averaged according to one of the alternatives illustrated in Fig. 19.11. Computation of gradient and aspect is directly related to the procedure for obtaining shading intensities for automatic hillshading of DTMs.

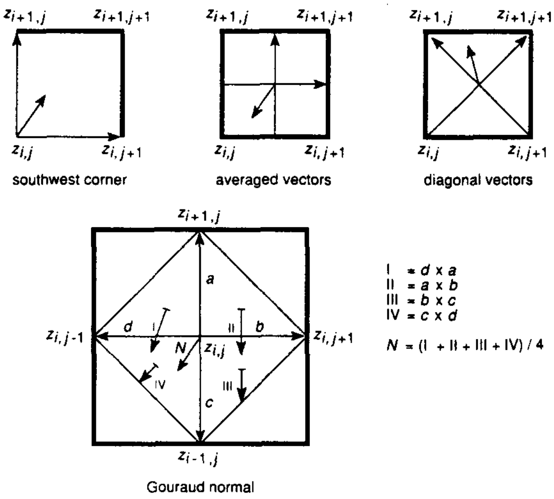


Fig. 19.11 Several alternatives to obtain averaged surface normals for square patches of either four or nine adjacent grid points.

A number of further parameters may be extracted from digital terrain data for geomorphological analysis. Mark (1975), Pike and Rozema (1975), Evans (1979), and Pike (1988) review various geomorphometric parameters and methods for their estimation: local relief (i.e. the range of altitude); hypsometric integral; drainage density; statistics of slope and convexity; parameters of the power spectrum; and others. The computation of these parameters is mainly based on gridded terrain data (Pike 1988; Weibel and DeLotto 1988). Parameter values are determined within a moving window of specified size (e.g. 11 × 11 grid points) with a technique comparable to that used in textural analysis in image processing (Haralick, Shanmugam and Dinstein 1973). Figure 19.10(d)) shows a map of the range of altitude for a moving window of 13 × 13. If this procedure is used, other parameters describing surface variation may be computed for the submatrix within the moving window, such as fractal dimension (Mandelbrot 1986; Roy, Gravel and Gauthier 1987) or entropy.

The above geomorphometric parameters may serve as input for multivariate classification of landforms (Pike 1988; Weibel and DeLotto 1988; Dikau 1989). DTMs may thus be used for applications such as landform analysis (Dikau 1989), classification of landslide hazards (Pike 1988), trafficability analysis, or as a means of supporting DTM generalization (Weibel 1989). Figure 19.12 shows the result of a multivariate classification of a DTM into regions of disparate terrain characteristics.

Specific geomorphometry

Techniques for specific geomorphometry have to date mainly focused on the delineation of terrain features related to surface hydrology. A range of features may be extracted from DTMs: surface-specific points (pits, peaks, passes, etc.); linear features (drainage channels and ridges); and areal features (drainage basins and hills). Definitions of these terms can be found for example in Douglas (1986). The objective of the analytical extraction process is to delineate the geometry of hydrological features, topologically connect them into contiguous networks, and obtain descriptive attributes for individual elements. This wealth of information may be used in applications such as hydrological runoff simulation (Band and Wood 1988; Band 1989), geomorphological modelling, support of interpolation procedures (Hutchinson 1988, 1989), or in cartographic generalization of DTMs (Weibel 1989).

Surface-specific point features may be identified by comparison of elevation differences in

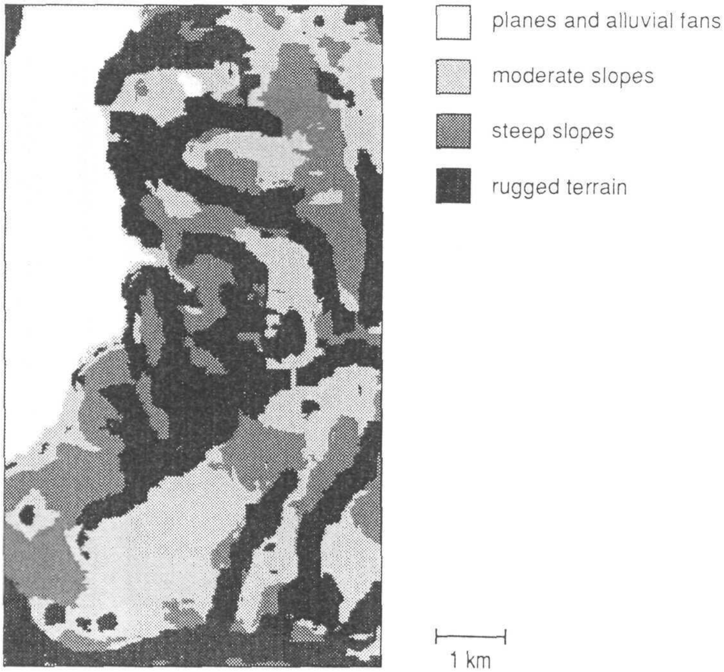


Fig. 19.12 Example of classification of a DTM (Fig. 19.10(a)) into regions of homogeneous terrain characteristics based on multivariate classification and subsequent region merging. (DTM data courtesy of Swiss Federal Office of Topography, 22.12.89)

a local neighbourhood (e.g. Peucker and Douglas 1975); peaks, for instance, are local elevation extremes. Algorithms for the extraction of linear hydrological features (channels and ridges) depend on the DTM data structure that is used. For gridded DTMs the so-called 'hydrological approach' (Mark 1984) is currently favoured by many authors (e.g. Marks, Dozier and Frew 1984; O'Callaghan and Mark 1984; Band 1989; Mark 1987). For a review and evaluation of other algorithms see Douglas (1986). In the hydrological approach, the drainage area of each grid point (i.e. the number of DTM points draining into that point) is first determined by climbing recursively through the DTM. The result of this operation is the so-called 'drainage area transform' (Band 1989), a matrix that contains the drainage area for all grid points. This information can subsequently be used to trace the channel pixels (i.e. those pixels with large drainage areas). Channels are recursively followed upstream until no further point can be found that exceeds a minimum drainage area threshold. The topology of

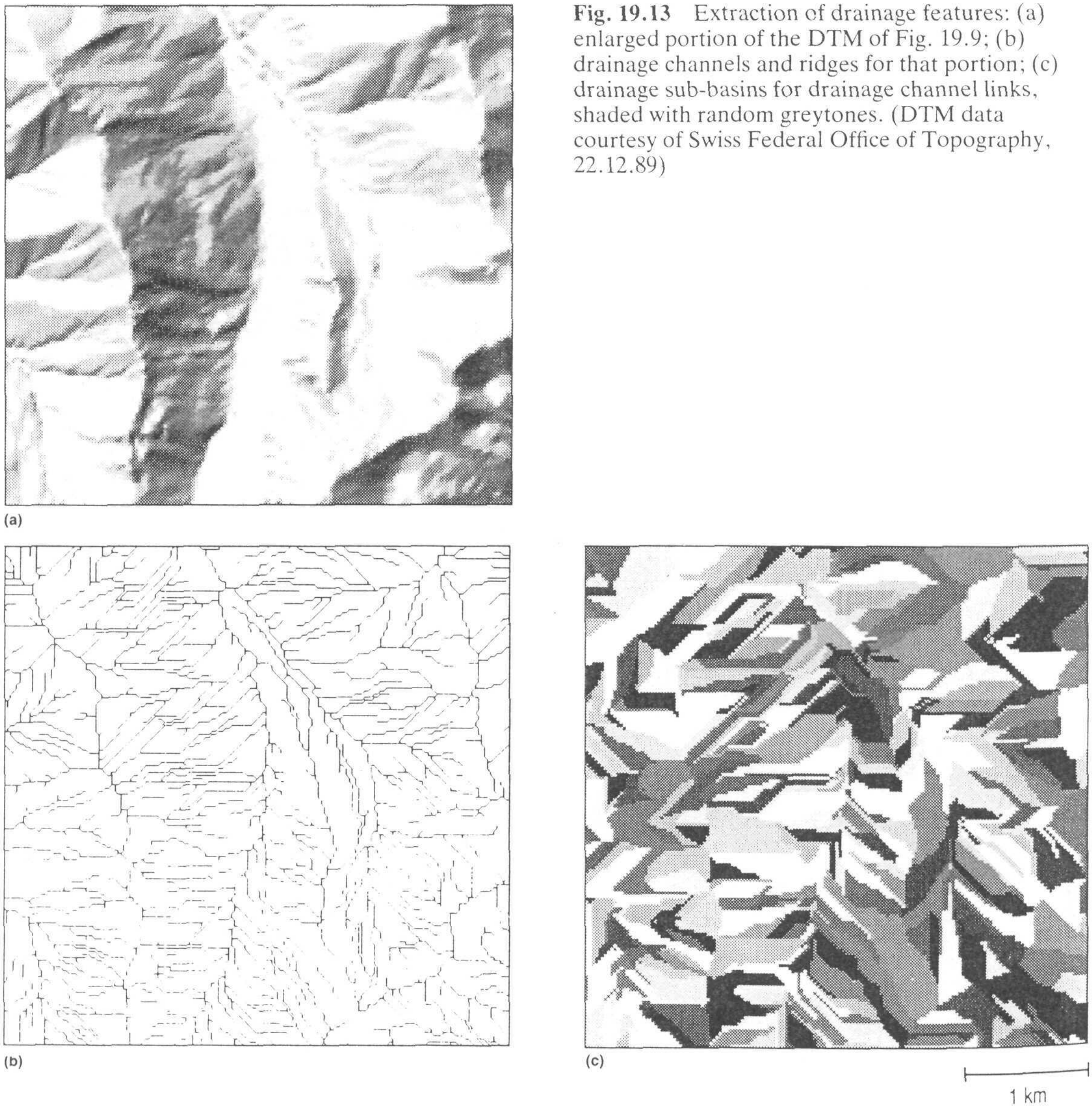


Fig. 19.13 Extraction of drainage features: (a) enlarged portion of the DTM of Fig. 19.9; (b) drainage channels and ridges for that portion; (c) drainage sub-basins for drainage channel links, shaded with random greytones. (DTM data courtesy of Swiss Federal Office of Topography, 22.12.89)

the channel network is formed concurrently with this trace operation. Ridges may be extracted by greyscale thinning of all non-stream pixels (Band 1986) or by delineating the boundaries of drainage basins. Figure 19.13(b) shows the complementary networks of channels and ridges that were identified by the above procedure.

The extraction of channels and ridges from TINs may be performed by vector analysis operations. For each triangle edge this may be determined from the direction of the surface normals of the two adjacent triangles. In a second more complex pass, channel and ridge edges are connected to contiguous topologically structured networks. A robust procedure should be able to enforce network connectivity even through artificial pits (caused by inadequate interpolation) or intermediate flat areas (e.g. lakes). Although most authors today use gridded DTMs for drainage feature extraction and apply the hydrological approach, it has to be noted that this method poses some obvious problems. Grids are a rigid sampling structure and cannot accurately represent discontinuities. The channels and ridges are forced to the locations of grid points although they would rarely pass through them in reality. Furthermore, some rather arbitrary operations (i.e. thresholding, greyscale thinning, etc.) are involved with that procedure.

Areal hydrological features (basins and hills) may be delineated using variants of the recursive DTM climbing or descending algorithms to find those grid points (or TIN facets, respectively) belonging to a specific channel or ridge link and thus constituting its sub-basin (Marks *et al.* 1984; Band 1989). Figure 19.13(c) shows the drainage sub-basins that relate to the stream channels of Fig. 19.13(b).

A number of topological attributes (e.g. link magnitude and topological order) and geomorphometric attributes (e.g. link length, mean slope and basin area) may be computed for individual links and their related sub-basins. Based on network topology, those attributes may also be aggregated for higher order links (e.g. for the root of a drainage basin). Topological models for tree-like networks are reviewed by Jarvis (1984). A useful model that combines channels and ridges into two interlocking networks has been presented by Werner (1988).

Interpretation for DTM quality assessment

Error detection and correction

Errors – blunders and constant, systematic and random errors – may occur in any sampling process (see Chrisman 1991 in this volume). Apart from geometrical errors, classification errors may occur (e.g. an edge in a DTM may be classified as part of a breakline when it is not). Procedures for detection and correction of errors are thus important.

The method most often used is visual inspection and interactive editing of the incorrect elements. Some display techniques are particularly useful for highlighting errors: on perspective displays, errors are detectable since they project out of the surface, whereas on hillshadings or maps of slope (or of other geomorphometric parameters) errors become obvious due to gradient anomalies. Apart from gradients, special error indices may be computed and visualized (Hannah 1981).

In conjunction with automated DTM extraction (by correlation of stereo images), there is a growing need for procedures for automated detection and correction of errors. Studies on reducing the amount of abnormal elevations at the time of DTM extraction have been reported by Nagao *et al.* (1988). A possible procedure for removing blunders in a post-processing step has been presented by Hannah (1981). In a first step, indicators of correctness are computed for individual DTM elevations (based on gradient differences around each DTM point). In a second step, an iterative procedure is employed to constrain errors to the elevation of adjacent points (based on correctness indicators).

DTM quality control

Quality control of a DTM may be performed by comparison with reference data (i.e. control points or another DTM). Parameters commonly used to evaluate the quality of a surface fit to reference points (e.g. Root Mean Square Error, RMSE), are reviewed by Willmott (1984). Other techniques are used for comparing two DTMs of the same area: statistical analysis of residual surfaces, comparative analysis of semi-variograms or frequency spectra. Procedures for DTM quality control may also be used to help detect systematic or constant errors. Day and Muller (1988) present a sample empirical study in which they assess the quality of DTMs produced by automatic stereo matchers.

Interpretation for planning and engineering applications

In addition to the tasks of geomorphometric analysis (mainly gradient and aspect mapping), planning and civil engineering applications require more specialized interpretation functions. One category of procedures is used for visibility analysis and for relief shadow analysis. The basic visibility algorithm for gridded DTMs determines the visibility of each point according to the schematic illustration of Fig. 19.14; an example of a visibility map is shown in Fig. 19.15. A more detailed description of this approach is given by Yoeli (1985a). A more efficient, recursive algorithm that obtains an approximate solution to the intervisibility problem has been proposed by Mark (1987). An algorithm to determine intervisibility within a TIN has been presented by De Floriani *et al.* (1986). Intervisibility may also be computed for multiple viewing points, by applying the visibility procedure for all viewing points and adding the results. The computation of relief shadows (Fig. 19.16) is closely related to visibility analysis (cf. Rogers 1985).

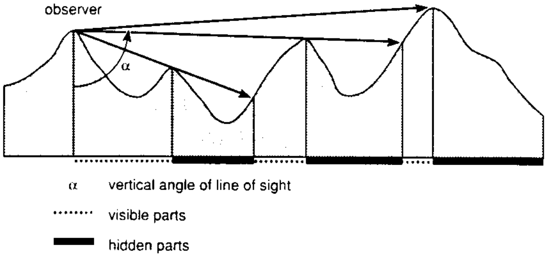


Fig. 19.14 Determining the visibility of a DTM point.

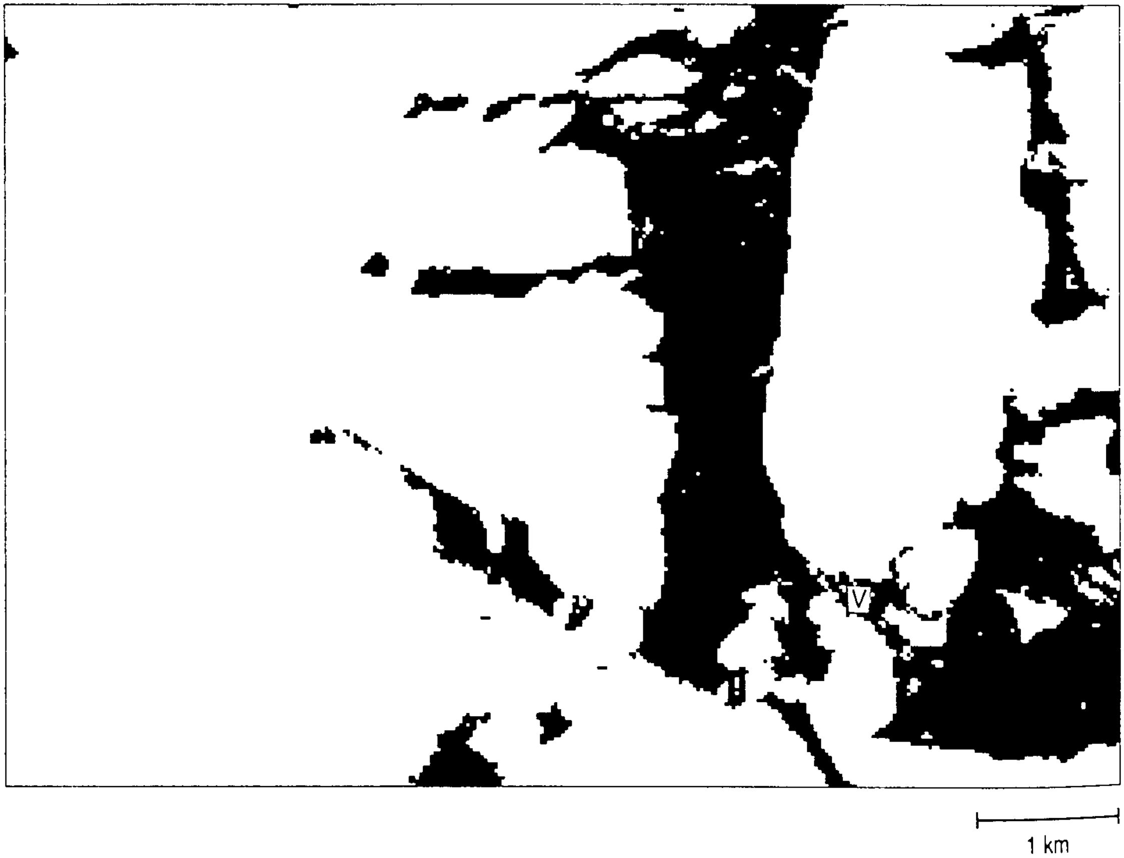


Fig. 19.15 Binary visibility map for the DTM of Fig. 19.9. Observer's location is at point V (same location as in Fig. 19.9). Visible parts are shown in black.

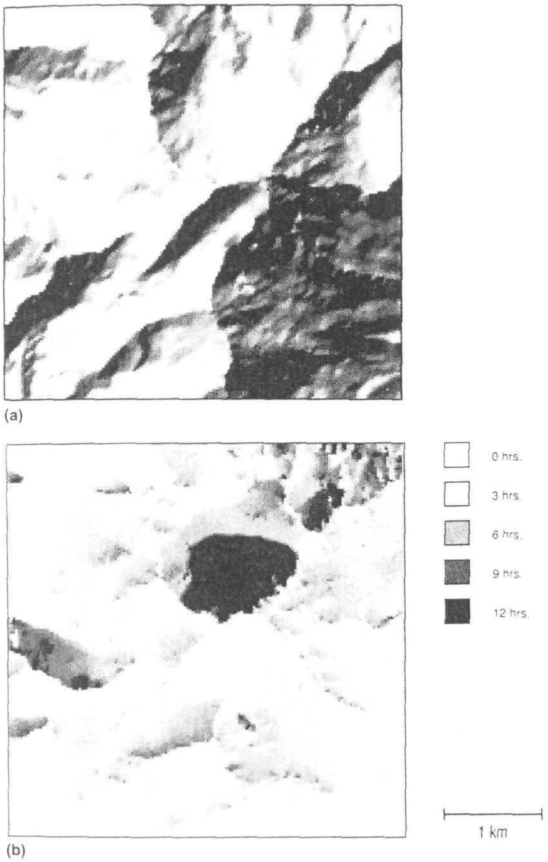


Fig. 19.16 Relief shadow analysis: (a) grid DTM (160 × 160 points); (b) shadow analysis shows how many hours each part of the DTM is in shadow; for 21 March, 6:00–18:00 MET (with V Antunes, FHS Karlsruhe). (DTM data courtesy of Swiss Federal Office of Topography, 22.12.89)

Instead of identifying the areas invisible from a given viewing location, the task is to determine the DTM elements that are obscured from a particular position of the sun (i.e. they lie in the shadow of other parts of the DTM).

Profile computation (cross-sections), and volumetric calculations (also called cut-and-fill) represent a second class of interpretation procedures for engineering and planning. These functions are used in applications such as road or reservoir design. Profiles are obtained by interpolating terrain heights at the intersection points of the profiles with the DTM. Volumetric computations are performed most easily on gridded DTMs. More accurate results can be achieved using TINs.

DTM VISUALIZATION: DISPLAYING DIGITAL TERRAIN MODELS AND RELATED DATA

Results of DTM (or GIS) modelling operations are most often communicated to users in graphical form (Buttenfield and Mackaness 1991 in this volume). Visualization thus plays a vital role in a DTM system. It is closely linked to interpretation: results of interpretation steps need to be displayed, and interpretation operations may in turn lead to improvements in visualization. Moreover, graphics themselves may directly support decision making (through visual interpretation) without involving any quantitative analysis.

Visualization commonly pursues two goals: interactive visualization, which helps the researcher to explore models and refine hypotheses; and static visualization (in the traditional form of paper maps), which is used to communicate results and concepts. The usefulness of visualization products mainly depends on their communicational effectiveness and their ability to support interpretation. In this light, utmost realism is not a primary objective.

Orthographic display techniques

Conventional forms of relief depiction (e.g. contours and hillshading) represent vast abstractions of reality, yet they are highly effective in solving many of the tasks related to the presentation and interpretation of terrain.

Contours

Contour lines (isolines) are probably still the most widely used technique for displaying relief. Contours represent a method for quantitative visualization of the third dimension. They are used to satisfy the requirement for extracting quantitative information from relief displays (e.g. in geology, topographic mapping and civil engineering). The major drawback of contours is that they give no immediate impression of the topographic forms.

The construction of contours (contouring) is closely related to DTM interpolation. A rich literature about contouring is available: many computer programs have been implemented for this task, and some of these developments have been documented in detail. Technical aspects of contouring TINs are for instance discussed by McCullagh (1981, 1988), and Gold and Cormack (1987); corresponding techniques for gridded DTMs are described by McCullagh (1981, 1988), Zyda (1988), among others. The best results are produced by methods that incorporate surface discontinuities into their contouring algorithm. Smooth contours may be generated by subdividing triangles or rectangular cells to yield a finer mesh for contour interpolation (e.g. McCullagh 1981, 1988). Alternative (more adaptive) methods produce smooth contours through contour following by successive solution of non-linear equations (e.g. Preusser 1984). Basic contouring may be further refined: index contours may be highlighted by special symbolization, contour labels added for index contours, and contour drawing may be suppressed in areas of steep gradient (i.e. where contours are too densely cluttered, only index contours are drawn).

A number of alternative methods exist for contour line display. Elevation data may simply be classified into equidistant altitude classes to produce a hypsometric tint display. Alternatively, gridded DTMs may be contoured by a technique called 'raster contouring': instead of threading contour lines through the grid, contours are determined on the basis of individual grid cells (Eyton 1984). Finally, two methods – called 'inclined contours' and 'relief or shaded contours' – may be used (Peucker 1972; Peucker and Cochrane 1974; Yoeli 1983). Both techniques attempt to extend contouring to give a better three-dimensional impression of relief. The first method represents intersections of the surface with parallel inclined planes that are orthogonal to the direction of the light source (i.e. inclined contour lines). The latter method draws contours by varying their width according to illumination brightness.

Hillshading

Analytical or automatic hillshading (Figs. 19.9, 19.10, 19.13 and 19.16) provides a convenient way of qualitative cartographic relief depiction. Landforms may be readily perceived from shaded relief displays. No clues are, however, provided about absolute terrain elevations. Hillshading for cartographic purposes was first automated by Yoeli (1965, 1967), and has become a standard terrain display technique since then.

The method is based on a model of illumination. Light intensities are computed for individual DTM facets, and facets are shaded accordingly. Shading values for DTM facets (i.e. light intensities) are computed in two steps. First, surface normals have to be determined for the individual facets (this sub-problem has already been discussed for slope analysis). Once the surface normal of a facet has been determined the illumination model is applied. The simplest model – Lambertian shading – assumes diffuse reflection of the illuminated object. The intensity of diffusely reflected light is proportional to the cosine of the angle between the surface normal and the illumination vector (i.e. the direction of the light source). And the cosine of that angle equals the dot product of the normalized illumination vector and the surface normal vector.

More sophisticated illumination models may be used. One technique, called Gouraud shading, interpolates smooth transitions of shadings between individual facets (this method has been applied to the block diagrams of Plates 19.3 to 19.4). Other advanced models (e.g. Phong shading and ray tracing) include further elements of illumination such as specular reflection, ambient light, transparency, or haze (cf. Plate 19.2(b)). A detailed discussion of illumination models may be found in Rogers (1985), or Foley *et al.*(1990). For the purpose of hillshading, however, it is most often sufficient to use a simple shading method.

Basic analytical hillshading may be further enhanced for cartographic purposes. Brassel (1974) has developed a model for automatic hillshading that attempts to emulate conventional cartographic principles. Vertical and horizontal adjustments of the light source are performed locally in order to enhance the depiction of landforms that run in the direction of the light source, and the general impression is improved by contrast enhancements. Yoeli (1985b) has attempted to automate relief depiction by hachures. However, although this traditional cartographic technique is quite artistic, it is probably not a very useful method for visualizing relief.

Combination with 2-D data, orthophotos

Contour displays as well as hillshading may be overlayed with other elements such as the results of interpretation procedures (e.g. visibility, slope and drainage networks); components of the DTM (e.g.

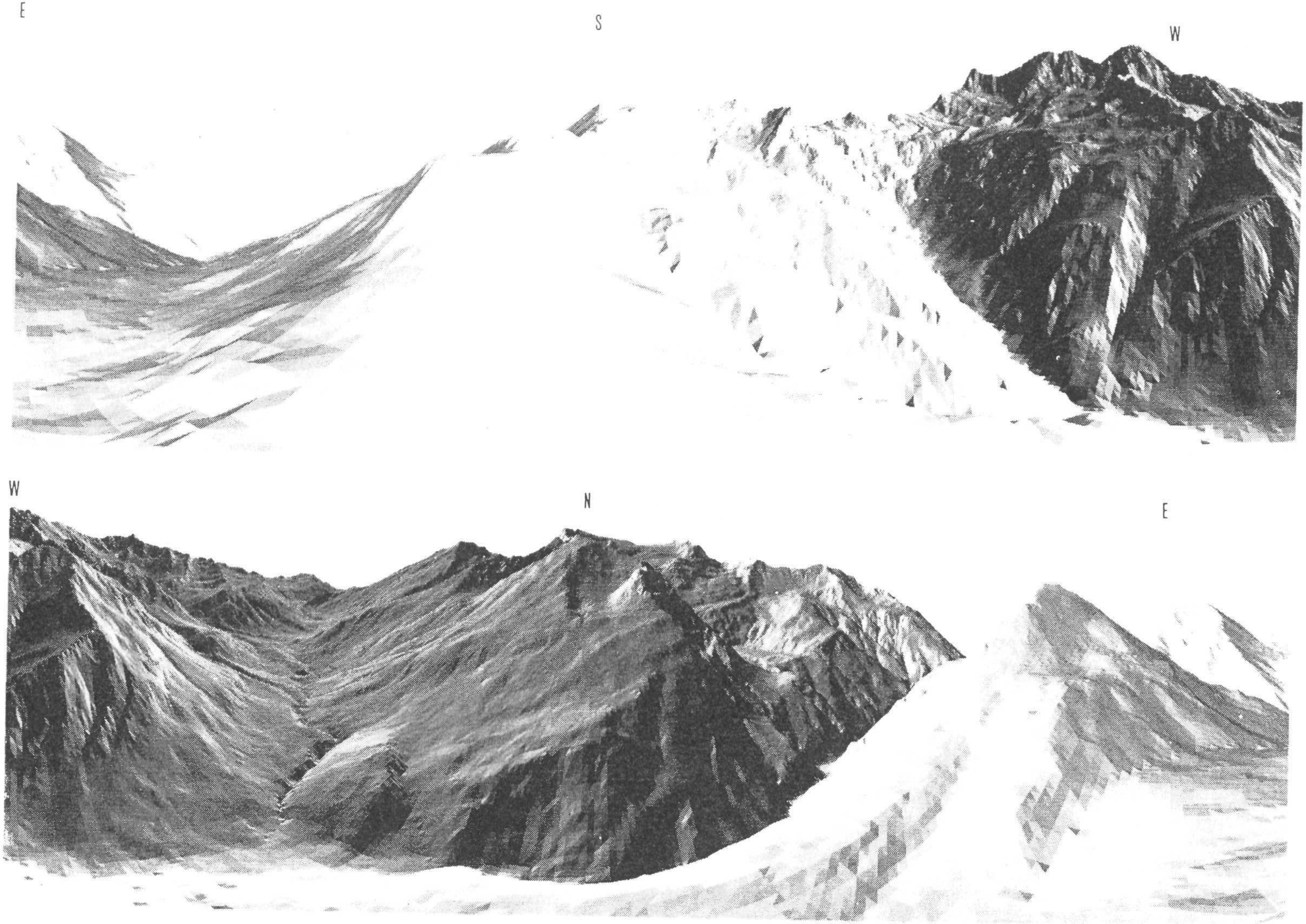


Fig. 19.17 Panoramic view of a DTM of the Simplon region (Valais, Switzerland) using simple shading. Compared to visibility maps panoramic views visualize the intervisibility situation in a more obvious way. (DTM data courtesy of Swiss Federal Office of Topography, 22.12.89)

data points, structural features and TIN triangles); or other 2-D data (e.g. roads, land use and geological maps). To combine areal data (e.g. land use) with hillshadings, the colour or intensity values of the areal data have to be modulated with the shading intensity at the corresponding location (for a description of this technique, see e.g. White 1985).

Orthophotos are a further orthographic display technique. They are generated from overlapping conventional aerial photos in a process called differential rectification (using DTMs) to eliminate image distortions due to topography (Slama, Theurer and Henriksen 1980). Orthophotos are commonly used in planning applications or topographic mapping.

Perspective display techniques

The advantage of orthographic displays is that all parts of the terrain surface are visible and relatively undistorted. Perspective displays, on the other hand, provide much more convincing visualization results (Plate 19.1). Two of the main problems that have to be solved for perspective display are the projection of the 3-D surface on to a 2-D medium, and the elimination of hidden elements from the display. The 3-D and perspective transformations (including 3-D clipping) necessary for perspective projection are described in standard textbooks on computer graphics (e.g. Newman and Sproull 1979; Foley *et al.* 1990).

As raster graphics devices (screens and plotters) have become predominant today, algorithms for hidden element removal are focusing on so-called image–space algorithms (i.e. the algorithm determines what is visible within each pixel of the raster display device). Algorithms may be further simplified for single-valued DTMs. Since no overlaps of DTM facets may occur in the depth dimension, this depth coherence may be exploited. For instance, a perspective block diagram may be generated by a series of vertical profiles. The corresponding procedure for removing hidden parts of profiles is the floating horizon algorithm (e.g. Rogers 1985). Instead of using line symbolization, the DTM may be displayed as shaded surface facets. Facets may either be shaded by applying a shading model as discussed for hillshading (Plate 19.1) or by using additional data such as land cover data or remote sensing images (Plates 19.2(a) and 19.2(b)). Algorithms for hidden surface removal are needed to generate displays of this kind. Backface elimination (Newman and Sproull 1979) may be used as a pre-processing step. This typically halves the number of facets that need consideration during hidden surface computation. Descriptions of the basic hidden surface algorithms (e.g. the painter's or the *z*-buffer algorithm, including related techniques such as anti-aliasing) can be found in standard textbooks (e.g. Newman and Sproull 1979; Foley *et al.* 1990; Rogers 1985). More sophisticated algorithms are discussed in relevant periodicals (e.g. those published by ACM SIGGRAPH, or IEEE).

Many of today's graphics workstations provide hardware implementations for perspective projection, as well as hidden element removal (mostly *z*-buffer), and object rendering. These functions greatly simplify the task of building highly interactive display and analysis systems that are capable of handling surface models of considerable size.

Panoramic views are a special variant of perspective displays: the observer is located on or near the surface, and most often the view extends over a full circle. This visualization technique provides a useful extension to visibility maps for planning applications (Fig. 19.17). The transformations for panoramic displays and their construction have been described by Herzog *et al.* (1987) and Weibel and Herzog (1988).

Advanced visualization techniques and issues

The basic forms of perspective block diagrams may be extended in a number of ways to support interactive visual interpretation and analysis of surface data. Cartographic features (point, line or areal data) may be overlaid on the DTM surface, place name labels added, or transparent surfaces used to indicate elevations (e.g. of a planned reservoir). Additionally, 3-D objects (e.g. houses, subsurface structures) may be displayed in conjunction with the block diagram. Sample visualizations from a system for interactive display and analysis of DTMs (cf. L'Eplattenier 1987; Herzog *et al.* 1987) are shown in Plates 19.3, 19.4 and 19.5). A more general system for the visualization of geoscientific data (including meteorological space–time data) has been presented by Hibbard and Santek (1989).

The evolution of terrain display is gradually

Functional requirements \ DTM application domains		Surveying & photogrammetry	Civil engineering	Planning & resource management					earth scienc.		Military
				Environm. & urban plann.	Remote sensing	Soil science	Agriculture & forestry	Meteorology & climatology	Geology & geomorphology	Hydrology	
DTM generation	ground survey data input	●	⬤	•							•
	photogrammetric input	⬤	⬤	●	•	•	●	•	•	●	•
	manual digitizing (contours)			⬤	●	⬤	⬤	⬤	⬤	⬤	●
	scanning/vectorization (contours)			•	●	•	•	•	•	•	⬤
	automatic correlation of stereo images	●		•	⬤		•				⬤
	use of existing (available) data	•	•	⬤	⬤	⬤	⬤	⬤	⬤	⬤	⬤
	non-terrain vars. form model input 1			•	•	•	•	⬤	⬤	•	•
	triangulation 2	⬤	⬤	●	●	•	●	•	⬤	⬤	●
	interpolation to gridded DTM	•	⬤	⬤	⬤	⬤	⬤	⬤	⬤	⬤	⬤
	interpolation from contour data			●	●	●	●	●	●	●	⬤
	special interpol. algs (non-terrain)			•	•	•	•	●	●	•	•
DTM manipulation	DTM editing	⬤	⬤	●	●	●	●	•	⬤	●	⬤
	DTM joining and merging	⬤	●	●	⬤	●	●	•	⬤	●	⬤
	TIN filtering	●	●	●	●	●	●	•	●	⬤	⬤
	grid-to-TIN conversion		●	●	●	•	•	•	⬤	⬤	⬤
	smoothing/enhancement filters		•	•	⬤	•	•	●	●	●	•
	obtain heights for 2-D features	●	●	•	•	•	•	•	•	●	●
	interface to raster system	•	⬤	⬤	⬤	⬤	⬤	⬤	⬤	⬤	⬤
DTM interpretation	autom. error detection/correction	●		•	⬤		•				⬤
	gradient/aspect analysis	•	●	⬤	⬤	⬤	⬤	•	⬤	⬤	⬤
	other derivatives (e.g. convexity)			●		•	•		●	●	●
	intervisibility analysis		●	⬤	•		•				⬤
	relief shadow analysis		●	●	•	•	•	•		●	●
	drainage network extraction			•	•	•	•		⬤	⬤	●
	drainage basin extraction			●	●	•	●		⬤	⬤	●
	profiles	⬤	⬤	●	•	●	•		●	•	
	volumetric calculations (cut & fill)	●	⬤	•	•		•		●		
DTM visulization	contouring (incl. hypsometric tints)	⬤	⬤	●	●	●	●	●	⬤	●	⬤
	automatic hillshading			●	●	•	•	•	⬤	•	⬤
	perspective (wireframe or shaded)	⬤	⬤	⬤	⬤	•	●	●	⬤	●	⬤
	perspective with 2-D overlays	•	●	⬤	⬤	•	•	•	●	●	⬤
	3-D objects on surface (perspective)		●	●					•		●
	orthophotos	●	●	●	●		●		●		●
	cartographic generalization			●	•	●	●	•	●	●	⬤
	photorealism		•	●							●
	animation			•							●
DTM applic	programming interfaces	●	●	●	●	●	●	●	●	●	●
	interfaces to exist. applic. progs.	⬤	⬤	⬤	⬤	⬤	●	⬤	⬤	⬤	●
Data 3 struct.	triangulated irregular network (TIN)	⬤	⬤	●	•	•	•		⬤	⬤	●
	rectangular grids (elevation matrices)	•	⬤	⬤	⬤	⬤	⬤	⬤	⬤	●	⬤
Data	data volume	⬤	●	⬤	⬤	●	⬤	●	⬤	●	⬤
	structural features	⬤	⬤	•	•	•	•	•	⬤	⬤	●
	statistical surfaces			•	•	•	•	⬤	⬤	•	•

Legend:

- ☐ not applicable or not typical
- • minor importance
- ● average importance
- ⬤ major importance

1 Examples of non-terrain variables: air temperature, population density, depth of geological horizon.

2 For two purposes: for TIN generation and as a basis for interpolation.

3 Data structures used for deriving DTM products.

Fig. 19.18 Some DTM application domains and their functional requirements relative to digital terrain modelling.

moving towards more realistic renderings of DTMs. Examples of photorealistic scene rendering may be found in Musgrave, Kolb and Mace (1989); Kaneda *et al.* (1989) or Upstill (1990). Musgrave, Kolb and Mace (1989) present an efficient algorithm for ray tracing height fields such as elevation matrices. Ray tracing is a rendering technique that allows the modelling of illumination effects such as reflections, refraction, transparency and shadows. Kaneda *et al.* (1989) discuss a method of terrain visualization for environmental assessment: digitized aerial photographs are mapped onto the DTM, and planned objects (e.g. power stations) may be added into the perspective display for a montage.

Displays of terrain surfaces may also be animated (e.g. for flight simulation or computer-generated films). In animation, sequences of scene renderings are produced that correspond to frames of a film (by moving the viewing point). In flight simulation, abstraction of the terrain surface may be quite dramatic (to speed up computation for real-time display), while other animations provide more realism (e.g. Muller *et al.* 1988).

Visualization combines two processes: modelling and selecting relevant elements for representation (scene description); and displaying them effectively (scene rendering). For the second task, it is possible to profit from the steady progress in computer graphics: standardized rendering interfaces such as 'RenderMan' (Upstill 1990) will increasingly allow scene descriptions to be passed to a turn-key rendering system. Research in the GIS area thus will mainly have to focus on methods for generating appropriate scene descriptions, while rendering systems will take care of the actual display.

One of the issues related to scene description is that of cartographic generalization (Muller 1991 in this volume). As more and larger data sets are being handled it is becoming more urgent to be able to automate adaptation of the detail of DTM visualizations to the display scale. Generalization requires complex tasks such as feature recognition, simplification, and visualization. Its automation can be considered one of the most demanding problems of geoprocessing. A review of some approaches to surface generalization (as well as for generalization of 2-D elements) is given by Brassel and Weibel (1988); specific research is reported in Weibel (1989).

DTM APPLICATION: USING DIGITAL TERRAIN DATA

Due to the recent technological advances, terrain modelling systems of increasing complexity are being implemented which offer powerful solutions to applications. It is becoming feasible to create specific applications based on this core functionality. Practical use will then provide feedback for further enhancements of DTM concepts and techniques.

This final section summarizes the particular functional requirements of various application domains. Figure 19.18 represents an attempt to estimate the importance of individual DTM functions to some fields. Weights that have been assigned are intended to reflect relative importance and general traits rather than absolute and indisputable figures. Also, it should be mentioned that functional requirements are likely to change over time, as research and development proceeds and new applications emerge.

Many of the applications in spatial data handling do not require (or only rarely require) the use of digital terrain models. This is notably the case for cadastral mapping and utilities management (an area often termed 'automated mapping/facilities management' = AM/FM). On the other hand, a great number of applications do have a need for digital terrain modelling, most importantly those related to the management of natural resources. Five main application domains of DTMs may be distinguished: (1) surveying and photogrammetry; (2) civil engineering; (3) planning and resource management; (4) earth sciences; and (5) military applications. These fields differ in scope and importance, and they are not always clearly discernable. However, they all show particular characteristics in terms of functional requirements, professional background, and organizational structure. About one-third of all applications in these domains make use of digital terrain models or derived products at some point.

Surveying and photogrammetry

Surveying and photogrammetry is a relatively limited field with quite narrow functional requirements. The main purpose is to produce

DTMs for applications in other areas such as civil engineering or planning. Emphasis is on functions to generate high fidelity DTMs, evaluate their accuracy (numerically and visually), and generate high quality cartographic contours. Little analytical functionality is required. Typical applications include: survey or photogrammetric terrain data capture; data quality assessment; terrain data editing; orthophoto production; and topographic mapping.

Civil engineering

DTMs are used in civil engineering for applications such as road design, airfield design and earthwork calculations in site planning (e.g. dams, reservoirs and open-cast mining). Civil engineering is an important user of terrain modelling. Functional requirements, however, differ from those of other GIS applications: emphasis is on volumetric calculations and design functions, and data volumes are typically much smaller (i.e. often only a few thousand data elements).

Planning and resource management

Planning and resource management is one of the major application domains of DTMs. The field combines applications from a number of relatively diverse disciplines, such as environmental and urban planning, remote sensing, soil science, agriculture, forestry, meteorology and climatology – all disciplines that are centred around the management of natural resources. Typical applications include environmental impact studies; industrial site location; the geometric and radiometric correction of remote sensing images; support of image classification in remote sensing by DTM derivatives; soil erosion potential models; crop suitability studies; development of harvesting strategies; wind flow and pollution dispersion models; and many more. The diversity of applications within this domain necessitates a wide range of functionality: powerful tools for interpretation, flexible visualization procedures, functions for data capture and verification, and support of TIN as well as raster data structures are needed. Another characteristic of planning and resource management applications is that they require a close link between the terrain modelling system and the 2-D functions of the GIS software used – especially with respect to polygon, network, and raster data processing.

Earth sciences

Applications in the earth sciences – geology, geomorphology, hydrology, and glaciology – are treated as a separate group, although they share many similarities with other applications in natural resources management. They require specific functions for modelling and interpretation of terrain discontinuities (most importantly the drainage and divide network). Many of the uses of DTMs within this application domain need a concise representation of fluvial terrain discontinuities. Sample applications include: drainage basin monitoring (e.g. for flood and pollution control); hydrological runoff modelling; geomorphological simulation and classification (e.g. simulation of drainage basin development); geological interpretation and mapping.

Military applications

Military applications combine aspects of all the other application domains. Terrain is one of the most important components of the military environment at the local or regional scale. Military agencies are significant producers of DTMs, and thus put considerable weight on functions for terrain data capture (including stereo correlation of DTMs and automatic scan digitizing of contours). Military uses of DTMs include site planning operations similar to those in civil engineering. Terrain analysis for battlefield management involves tasks such as intervisibility analysis and vehicle trafficability analysis. Missile guidance and planning of communication networks (through intervisibility analysis) are other typical military uses of DTMs. Military applications also require advanced visualization functions such as photorealistic scene display and animation for flight simulation.

CONCLUSION

This chapter has sought to demonstrate the importance of Digital Terrain Models (DTMs) in many GIS applications. The discussion has outlined the scope of digital terrain modelling which encompasses the tasks of DTM generation; manipulation; interpretation; visualization; and application. Each of these processes has been examined with the discussion focusing on the relevant approaches adopted and algorithms used. At all stages, the need to combine mathematical and algorithmic approaches with environmental and, especially, geomorphological understanding has been highlighted.

In the early 1990s digital terrain modelling has reached the stage that some of its requirements have been satisfied. The agenda for the next few years includes the need for refinement of current techniques as well as the enlargement of their scope in order to handle increasingly complex DTMs. Developments in digital terrain modelling will be accelerated by general technological and scientific advances, and likely by synergy effects with parallel disciplines.

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