"Magnifying-Glass" Azimuthal Map Projections

John P. Snyder

ABSTRACT. For maps focusing on a region of interest, but including surrounding areas to provide a setting, new azimuthal projections have been developed with a "magnifying-glass" effect. On two such projections, inside a circle bounding the region of interest is a standard Azimuthal Equidistant or Lambert Azimuthal Equal-Area projection. Between this circle and an outer bounding circle, azimuths remain true and the radial or area scale, respectively, remains constant, but at a reduced value. On four other projections, the inner portion is a standard azimuthal projection, which may be Stereographic, Gnomonic, or the above, but beyond this portion, the radial scale is gradually reduced to zero. Equivalents with rectangular boundaries are also available.

KEY WORDS: Map projection, azimuthal projection

Most maps are designed to display all regions contained within their neatlines with as little relative distortion as possible, considering the various problems of map projection. However, there also are requirements for maps focusing on a region of interest, but including surrounding areas to provide a setting for the central region.

The Orthographic projection, perspectively projecting the Earth from infinity, with the map extending normally to a full hemisphere centered on the region of interest, is probably the best known of projections used for the latter requirements. If the General Vertical Perspective projection is used instead, with the viewer at an arbitrary height above the Earth's surface on the near or far side, the horizon may be established at any limit more or less than a hemisphere. The rate at which the radial scale falls off from the scale at the center is still more flexible when using a logarithmic or other specially designed azimuthal projection (Hägerstrand 1957; Tobler 1962, 1963). One form of a logarithmic azimuthal has a radius

$$\rho = R_b \ln(1 + sz)/\ln(1 + sb) \qquad (1)$$

where ρ is the radial distance from the center to a point at angular great-circle distance z from the center, R_b is the radius of the outer circle bounding the map, b is the map range up to 180° in the same units as z (degrees or radians), s is an arbitrary constant chosen to obtain the desired enlargement near the center, and ln is the natural logarithm. As s approaches zero, the plot approaches an Azimuthal Equidistant projection.

The above projections are artistically appealing, or have been used for thematic studies, but they have a constantly varying distance and area scale along any radius. Several new projections have been devised with a "magnifying-glass" effect. For the first two, the principal properties of a well-known projection, the Azimuthal Equidistant or the Lambert Azimuthal Equal-Area, respectively, are preserved for an inner and an outer portion with an abrupt change of scale at the boundary. For another four, the inner portion is a standard azimuthal projection, but the radial scale beyond this region is gradually reduced to zero at the chosen outer limit. These projections relate to circular magnifying glasses, but the principles may also be related to rectangular magnifying glasses, with somewhat greater complexity.

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THE "MAGNIFYING-GLASS" AZIMUTHAL EQUIDISTANT PROJECTION

The first projection, called the "Magnifying-Glass" Azimuthal Equidistant (Figure 1), is a true Azimuthal Equidistant projection within a prescribed radius of the center. For this portion of the map, the distance measured radially on this map from the center of the projection to any other point is the same as the distance on the sphere at the stated map scale. Beyond the circle bounding this portion, the radial scale is reduced to a constant fraction of the stated map scale until the outer limit of the map is reached. The map may include the entire Earth. The azimuths from center remain correct throughout the entire map, making the projection truly azimuthal. While a true magnifying glass hides part of the map surrounding the enlarged region, this projection shows the map without interruption. Due to the sudden change in scale at the inner limit, it is recommended that the inner circle always be plotted.

The radial scale pattern is shown in Figure 2. The radius providing this feature is computed as follows: Let

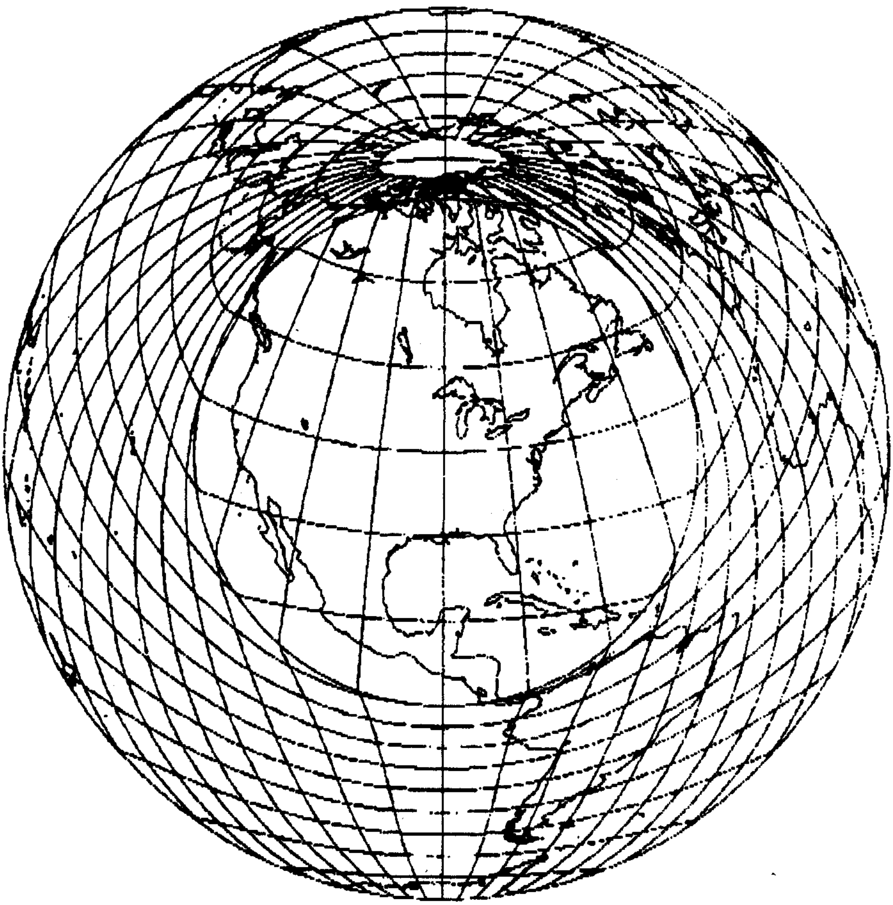

Figure 1. "Magnifying-Glass" Azimuthal Equidistant projection centered at 40° N., 90° W. with the inner circle at 30° radius, the outer circle at 120° radius, and the radial scale factor g_r between these circles set at 0.25.

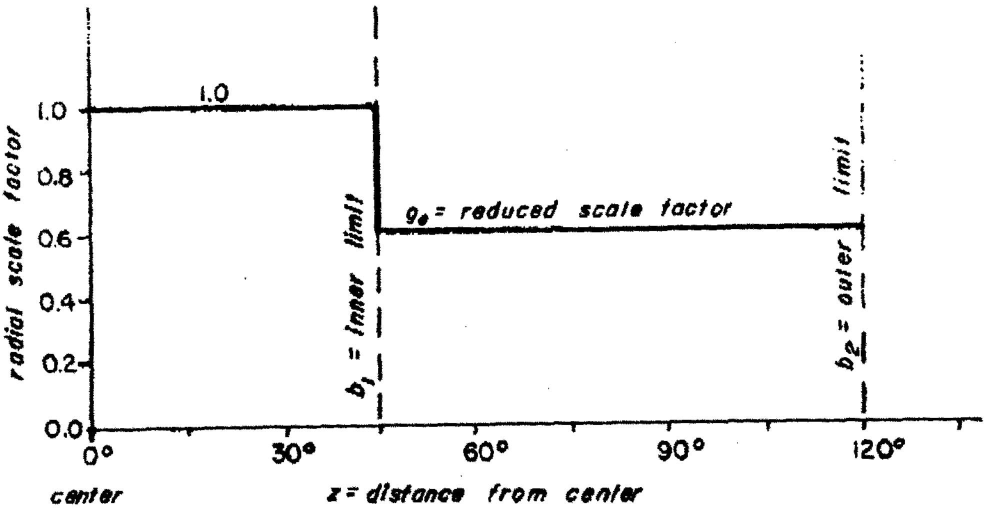

Figure 2. Scale factor as a function of distance from center for a typical "Magnifying-Glass" Azimuthal Equidistant projection.

$$C_e = b_1 (1 - g_e) + b_2 g_e \qquad (2)$$

If $z \leq b_1$,

$$\rho = R_b z / C_e \qquad (3)$$

If $z > b_1$,

$$\rho = R_b [b_1 (1 - g_e) + g_e z] / C_e \qquad (4)$$

where ρ, z, and R_b are as in equation (1), b_1 is the value of z for the inner map boundary, b_2 is the value of z for the outer boundary, and g_e, normally less than 1.0, is the radial scale factor between b_1 and b_2. The same units (degrees or radians) must be used for z, b_1 and b_2, with $b_2 \leq 180°$ and $b_1 < b_2$. If g_e equals 1.0, the entire map is an ordinary Azimuthal Equidistant projection. If g_e is greater than 1.0, the central portion is reduced instead of enlarged. To relate R_b to the radius R of the globe,

$$R_b = R\, C_e \qquad (5)$$

The radius r of the inner circle,

$$r = R\, b_1 \qquad (6)$$

For a given ratio n of radii (r/R_b),

$$g_e = b_1 (1 - n) / [n (b_2 - b_1)] \qquad (7)$$

and

$$C_e = b_1 / n \qquad (8)$$

THE "MAGNIFYING-GLASS" AZIMUTHAL EQUAL-AREA PROJECTION

The second projection is called the "Magnifying-Glass" Azimuthal Equal-Area (Figure 3) and often is not easily distinguishable from the first projection for the same ranges. Its inner portion is a true Lambert Azimuthal Equal-Area projection, with a constant area scale equal to the square of the stated linear map scale. Beyond the inner circle, the area scale factor is constant at a reduced value. Figure 2 applies to this projection as well, except that g_e becomes g_a, the reduced area scale factor, and the ordinate of the graph is the area scale factor. The formulas are as follows:

Let

$$C_a = [1 - (1 - g_a) \cos b_1 - g_a \cos b_2]^{1/2} \qquad (9)$$

If $z \leq b_1$,

$$\rho = R_b (1 - \cos z)^{1/2} / C_a \qquad (10)$$

If $z > b_1$,

$$\rho = R_b [1 - (1 - g_a) \cos b_1 - g_a \cos z]^{1/2} / C_a \qquad (11)$$

where all symbols and comments above apply, except that g_a is an area scale factor. To compute the outer radius,

$$R_b = 2^{1/2} R\, C_a \qquad (12)$$

Figure 3. "Magnifying Glass" Azimuthal Equal-Area projection, centered at 45° N., 85° W., with the inner circle at 7° radius, the outer circle at 20° radius, and the area scale factor g_s between the inner and outer circles set at 0.25.

For the inner-circle radius r,

$$r = 2^{1/2} R\,(1 - \cos b_1)^{1/2} \qquad (13)$$

For a given ratio n of radii (r/R_s),

$$g_s = (1 - n^2)\,(1 - \cos b_1)/[n^2\,(\cos b_1 - \cos b_2)] \qquad (14)$$

and

$$C_s = (1 - \cos b_1)^{1/2}/n \qquad (15)$$

THE "MAGNIFYING-GLASS" TAPERED AZIMUTHAL PROJECTIONS

The remaining new projections of this type for circular regions, called "Magnifying-Glass" Tapered Azimuthal projections, differ from the first two in that only the inner portion of each has the basic property of a standard azimuthal projection. The radial scale factor for the outer portion gradually changes from the radial scale factor at the outer edge of the inner portion to zero at the outer circle. The scale pattern for the Azimuthal Equidistant form is shown in Figure 4. The shape of the tapering curve depends both upon the spherical distance between b_1 and b_2 and the desired spacing between their circles on the map. These projections at first glance may appear to be Orthographic or another perspective view of the globe,

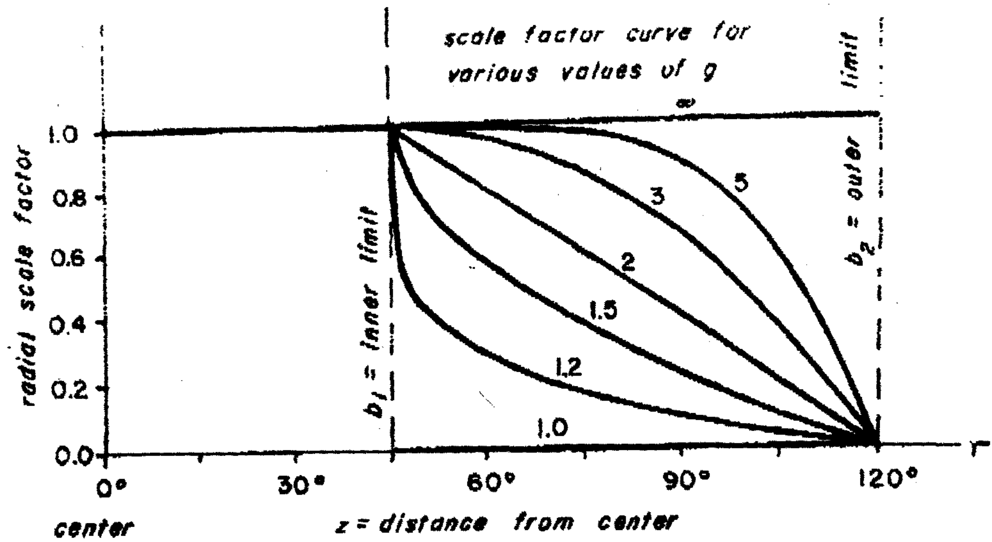

Figure 4. Scale factor as a function of distance from center for sample "Magnifying-Glass" Tapered Azimuthal Equidistant projections.

depending on parameters, but the inner portion is a true azimuthal projection of another type.

To shorten the names, they may be called, respectively, (I) the Tapered Azimuthal Equidistant (Figure 5), (II) the Tapered Azimuthal Equal-Area, (III) the Tapered Stereographic, and (IV) the Tapered Gnomonic projection. The appearance is affected more by the change in limits and the relative spacing between them than by the choice of inner projection, except for larger inner regions. The property of (I) constant radial scale, (II) constant area scale, (III) conformality, or (IV) straight great-circle routes, respectively, is maintained within the inner circle, but it is lost outside this circle. Figure 4 qualitatively applies to the last three, except the radial scale from the center to the inner limit is not constant, but varies in accordance with the particular projection, and the tapering curve varies in shape for the same reason. The function used for the tapered radial scale factor h' is as follows, if $z > b_1$:

$$h' = A\{1 - [(z - b_1)/(b_2 - b_1)]^{g-1}\} \qquad (16)$$

where A is the radial scale factor at b_1 for the particular inner projection, and g is an arbitrary constant determined below. This integrates to give a radius for the outer region as follows, after applying an integration constant to provide a fit between inner and outer projections:

$$\rho = (R_b/C)\{G + z - b_1 - (b_2 - b_1)[(z - b_1)/(b_2 - b_1)]^g/g\} \qquad (17)$$

$$\text{where } C = G + (b_2 - b_1)(1 - 1/g) \qquad (18)$$

and for the four projections, using the Roman numerals above,

$R_b = R\,C$	(I)	(19)
$= R\,C\cos(b_1/2)$	(II)	(20)
$= R\,C/\cos^2(b_1/2)$	(III)	(21)
$= R\,C/\cos^2 b_1$	(IV)	(22)
$G = b_1$	(I)	(23)
$= 2\tan(b_1/2)$	(II)	(24)
$= \sin b_1$	(III)	(25)
$= \sin b_1 \cos b_1$	(IV)	(26)

For the inner region, with $z \le b_1$,

$\rho = R\,z$	(I)	(27)
$= 2\,R\sin(z/2)$	(II)	(28)
$= 2\,R\tan(z/2)$	(III)	(29)
$= R\tan z$	(IV)	(30)

and r is the same as ρ in equations (27) through (30) with b_1 substituted for z. Symbols R, R_b, r, ρ, b_1, b_2, and z are previously defined. Radians must be used for angles in several (preferably all) of

Figure 5. "Magnifying-Glass" Tapered Azimuthal Equidistant projection with the same parameters as Figure 1, except that the ratio of radii of inner to outer circles is 0.65.

the equations (17) through (32). To obtain a given ratio n of radii (r/R_b),

$$g = (b_2 - b_1)/[b_2 - b_1 + (1 - 1/n)G] \quad (31)$$

However, n must be less than 1 and greater than n_0, where

$$n_0 = G/(G + b_2 - b_1) \quad (32)$$

If n is very near n_0, the outer region has almost no tapering, but approaches a constant radial scale factor, while the inner region does not appear to be enlarged. As Figure 4 shows, lower values of g result in more rapid reduction of scale near the inner circle.

RECTANGULAR "MAGNIFYING-GLASS" AZIMUTHAL PROJECTIONS

If the map is to emphasize a rectangular rather than a circular region, the six projections may be redeveloped so that an inner rectangle of any proportions (such as a square) contains the standard azimuthal projection, and the scale is reduced to another constant or in a tapered manner between the inner and outer rectangles (both having the same proportions). The outer region is also azimuthal, but general scale relationships are functions of azimuth as well as distance from the center. The geometry is somewhat more complicated, but the product may be useful (Figure 6). The formulas are omitted here.

GENERAL COMMENTS

To convert ρ in any of the above equations, or for any azimuthal projection, to rectangular coordinates, the following equations apply for polar, oblique, or equatorial aspects:

$$x = (\rho/\sin z) \cos \phi \sin (\lambda - \lambda_0) \quad (33)$$

$$y = (\rho/\sin z) [\cos \phi_1 \sin \phi - \sin \phi_1 \cos \phi \cos (\lambda - \lambda_0)] \quad (34)$$

where

$$\cos z = \sin \phi_1 \sin \phi + \cos \phi_1 \cos \phi \cos (\lambda - \lambda_0) \quad (35)$$

If z is 0° or 180°, equations (33) and (34) are indeterminate in this form, but x and y are zero for $z = 0°$, and if z is 180° the point is represented by the outer circle for the circular projections being discussed here. The origin of coordinates is at the center of the projection (lat. ϕ_1, long. λ_0), and the Y axis coincides with the central meridian λ_0, y increasing northerly. The point to be plotted is at lat. ϕ, long. λ. Longitude increases easterly.

The principle involved in the first two projections can be extended if desired to the preparation of maps with three or more steps of linear or area scale. This principle of a fixed reduced scale cannot be extended to the Stereographic and Gnomonic projections, and still maintain conformality or straight great-circle lines in the outer region.

The first two projections, however, achieve retention of important map properties of radial scale or area at the same time that a special region of concern is enlarged. The next four projections have a more esthetic transition into the outer region, but permit the inner region to be treated with precision

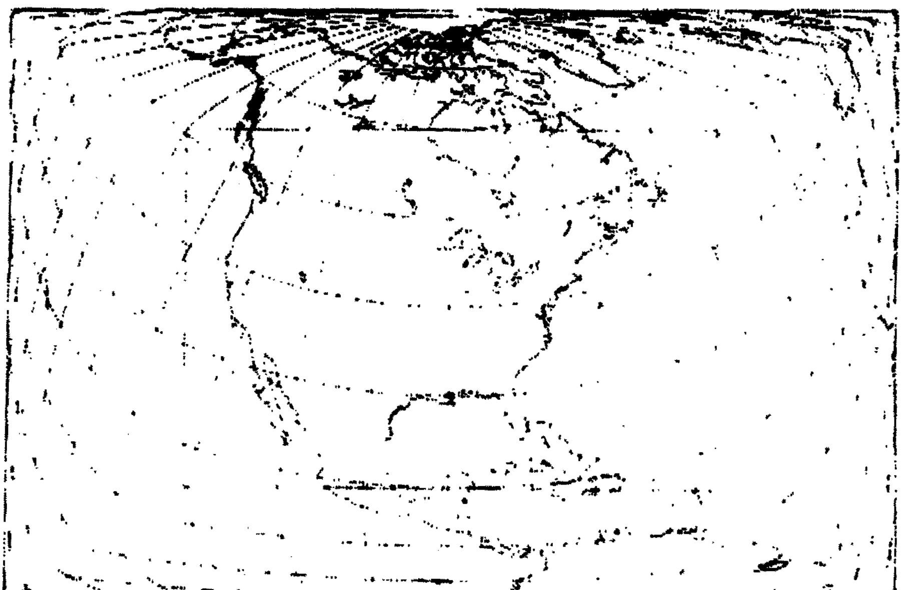

Figure 6. Rectangular "Magnifying-Glass" Tapered Azimuthal Equidistant projection centered at 40° N., 90° W. with the inner rectangle extending 30° east and west of center, and 20° north and south. The outer rectangle extends 90° and 60°, respectively. The inner rectangle has 0.6 the plotted length and width of the outer rectangle.

because of definitive properties for which long-used azimuthals are well known. The rectangular projections described have corresponding properties.

News media frequently display a map of a local region with an inset map of the country or continent containing and pinpointing the local region. The "magnifying-glass" projections would often permit the combining of the local and general regions into one map with the relative scales adjusted to suit. Another potential use is at a conference held to discuss, for example, the effects of an earthquake or storm. In this case, one of these new projections may be useful both for visually locating the sites and for determining relative distances or areas, without shifting the attention of the audience from one map to another. Because the projection range is flexible, any of these projections may be used for a region larger than a continent or as small as a town surrounded by the rest of a state or nation.

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