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SCALE-DEPENDENCE AND SELF-SIMILARITY IN CARTOGRAPHIC LINES

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INTRODUCTION

A digital cartographic object is not the same as the geographic feature it represents. That is to say, the graphic representation may progress through structural and visual variation as it is depicted at finer levels of resolution. In the past when base maps were compiled manually this was not a problem, because a different compilation was produced for each required map scale. However, use of a single digital base map file for depicting features at several map scales brings to attention both consistency and recognizability of base map information as it is generalized.

Visual cues are evident in a graphic depiction that map readers use to identify one feature as a river, another as a road, or a third as a political boundary. Some cues, such as color do not have a direct geometric component. Other visual cues may be measurable as the geometric details evident along the extent of a feature. These details may also be used to identify the scale of graphic depiction, in the context of geomorphic or photogrammetric applications (Goodchild 1980). Cartographically speaking, it is essential to retain both the details required for geographical accuracy and required for recognizability within a digital data base. This is the basis of map generalization, and the task is accomplished intuitively, when done by hand, and is based upon visual logic ('this solution looks right').

For computer mapping or analysis of GIS map information the issue becomes more complex, as analytic computations are performed on the digital version stored on disk, and not necessarily on the version displayed on the CRT screen. The graphic depiction seen by a GIS user may appear at a different scale than the digital version, and may contain only a subset of the coordinates stored on disk. Rules to determine which subset of detail to incorporate into a given scale of feature depiction must be explicitly defined, and incorporated into our display software automatically, to preserve consistency and accuracy for GIS displays. Brassel and Weibel (1988) apply the term 'statistical generalization' to operations of data reduction and filtering of the stored information, and the term 'cartographic generalization' to tasks of data display. In their conceptual framework, statistical generalization focuses upon the stored digital version, and

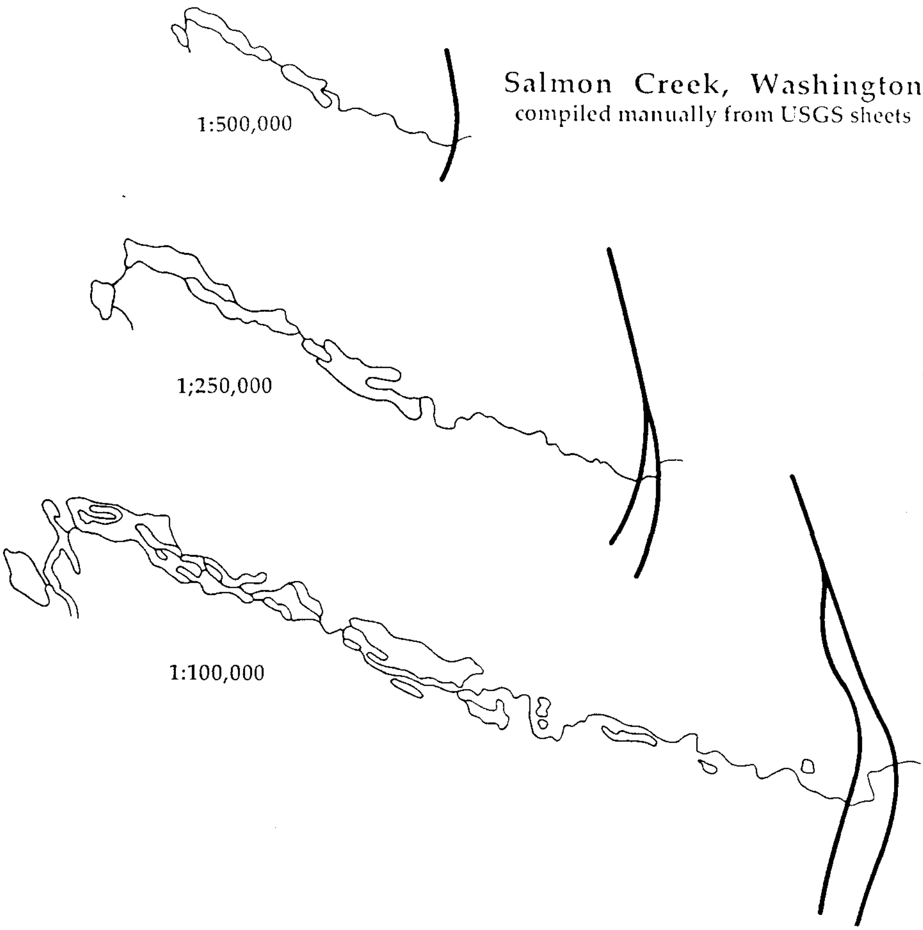

FIGURE 1 A portion of the Salmon Creek, near Vancouver, Washington, compiled by hand from USGS topographic sheets. The heavier weight line represents Interstate 5, running North-South across the river channel.

emphasizes positional accuracy. Cartographic generalization focuses on the displayed version, and emphasizes what they refer to as visual effectiveness, or recognizability.

To preserve accuracy and recognizability automatically during map generalization, one must be able to describe digitally the details that must be preserved. Digital encoding of the type of details stored in a coordinate file may provide more consistent guidelines for both statistical and cartographic generalization, and reduce problems in subsequent analytical tasks, such as edge matching across map sheets. The difficulty is that graphic details may vary distinctly from one scale of depiction to the next, and from one line feature to the next. It is probable that a single digital description model cannot appropriately describe all types of cartographic details that may be encoded for map representation.

Figure 1 illustrates a section of the Salmon Creek in Washington state, as compiled on a series of topographic maps. Over a range of map scales, river details are not simply replicated and enlarged. New details including braided channels and small islands appear at larger scales. Each compilation of the river contains only a subset of the details in the geographic feature, and the subset changes with the scale of graphic depiction. The highway, on the other hand, is a cultural feature, constructed according to fixed geometric constraints (e.g., a fixed radius of curvature for bends in the road). This feature behaves quite differently than the river, and its details do not change radically from one map scale to the next.

In light of this example, consider two classes of geometric objects. The first class may be characterized by self-similarity, meaning that structural detail is replicated through changes in scale. A special case of this class may be called scale-free geometry, as shown by the road in Figure 1: for a scale-free object, detail is preserved regardless of scale or resolution. The second class of objects may be called scale-dependent, and is characterized by distinct changes in structure with changing map scale, as shown by the river in Figure 1. 'Structure' is used here in the allometric sense (Gould 1966) referring to smaller pieces of a larger whole. For self-similar objects, the pieces are either statistically or precisely similar to the whole (Mandelbrot 1982). For scale-dependence, the pieces differ depending upon their scale of graphic depiction.

Studies of scale and structure may be found in many disciplines (e.g., Thompson 1961; Richardson 1961; Gould 1966; Stevens 1974; Spencer-Brown 1979). The common observation in all relates to the varying rates at which structural change may be observed at finer resolutions of measurement. Recent work by Mandelbrot has referred to self-affinity (Mandelbrot 1986) as a broader concept than strict self-similarity. Carpenter (1980) argued that self-similarity is most often encountered only within a finite range of resolution, and Mandelbrot appears to have adopted a similar though not equivalent view. This paper extends the Carpenter assertion and argues that naturally-occurring features display scale-dependent properties at specific and identifiable levels of resolution, and that the levels at which scale-dependence becomes apparent vary from one feature to the next.

The digital encoding of map features and subsequent computer representation at many levels of resolution raises challenging questions. Is it possible to produce multiple graphical representations from a single digital file, and preserve both accuracy and recognizability? Over what range of scales may a single data base by applied? How may digital encoding be designed to accommodate those features whose structure changes with scale, while also constraining other features in the data base whose structure does not change? And finally, without attempting to formalize the rules for generalization, can we at least formalize the types of geometric structure one can expect to encounter in representing features on maps? This paper addresses these questions, proposes a digital method for determining differences in feature geometry, and demonstrates its application empirically for a set of cartographic line features.

THEORETICAL ASPECTS OF THE PROBLEM

The definition of geometry formalized by Felix Klein deals with properties of features that remain invariant under certain classes of transformations. This definition requires a broader perspective of structure than is provided by the five Euclidean postulates, and accommodates more realistic descriptions of naturally-occurring phenomena. It was developed in response to the challenge of empirical verification of Euclid's parallel postulate (Greenberg 1973; Yaglom 1962). Euclidean geometry stresses form and structure in stasis. In Euclidean space, for example, the distance between two parallel lines is constant. Klein coined the terms 'hyperbolic geometry' and 'elliptic geometry' for spaces in which the distance between parallel lines respectively increases or decreases. For example, our perception of the size of an object will vary as we move toward it; however, rotating the object will most likely not affect our estimate of its size at a given distance. (This is somewhat of an oversimplification but will serve for the purpose of discussion.) Klein's goal of formalizing more realistic descriptions of structure requires distinguishing between operations that modify an object's structure and operations that preserve it.

Cartographic transformations are often defined in the context of preserving particular geometric characteristics, most commonly shape or relative size, as in the context of spherical map projections. In some cases, cartographic transformation is intended to preserve relations between spatial attributes and spatial structure (e.g., geodetic control, in rubber sheeting, or population counts, in cartogram construction). Less conventional is a consideration of map generalization as a cartographic transformation. Line simplification, for example, may be approached as a transformation preserving positional accuracy or recognizable details in changing the scale of a cartographic representation. In a metaphorical sense, the geometric components in the transformation are measurable as changes in line length or total angular change, and these form the basis for evaluation of simplification algorithms (McMaster 1983).

The work of Attneave (1954) demonstrates the relation between geometric measures (particularly angular change) and recognizability of linear detail. One may also pursue the transformation metaphor in terms of distance, defining simplification as a transformation that minimizes the number of coordinates while retaining the maximum overall line length (preserving the greatest amount of detail using the fewest coordinates). Previous authors have applied optimization principles to line simplification, by minimizing both coordinates and vector displacement (Lang 1970; Deveau 1985; Cromley and Morse 1988), or by arguing for minimizing both coordinates and processing time (McMaster 1988). These are valid objectives. The presumption in every case is that structural detail is homogeneous for all types of features, regardless of scale, and that simplification can be accomplished by uniform application of objective constraints along the full extent of a digital file. Uniform application of generalization constraints has been previously commented upon:

> This approach to cartography is rather mechanical....The cartographic tradition is more artistic and imaginative; strict adherence to positional accuracy is abandoned in favor of an approach which judiciously over-enlarges certain features. Consider the artisan producing smaller and smaller scale models of an automobile. If he attempts to adhere on a strict scale relation, but must eliminate features of less than a certain size, his model may eventually no longer look like an automobile. His objective is, however, that the model

continues to be recognizable as, and to have the attributes of, an automobile. Somehow he must capture the essential characteristics of automobiles, as a class of objects, and preserve these characteristics during the reduction of scale. The objective of cartographic generalization appears to be similar. TOBLER, 1964, p. 1

It has been demonstrated empirically that different simplification algorithms will produce versions of a cartographic line that differ in terms of angular, areal, and distance-based measures (McMaster 1987a). Algorithms are in fact relatively easy to categorize in terms of self-similarity and scale-dependence. For example, in applying the Radical Law (Topfer and Pillewizer 1966), assumptions of homogeneous detail are coupled with the equiprobable importance of each coordinate. The Radical Law assumes self-similarity in simplification. The rate of coordinate reduction is constant for scale reduction from 1:62,500 to 1:250,000 or from 1:10,000,000 to 1:40,000,000, regardless of the differing geomorphic structures that are visually evident across one range of map scale but not the other. For example, evidence of isostatic rebound may be apparent in a small-scale (1:40,000,000) representation of Hudson Bay; but at much larger scales, nearer 1:100,000, erosional features may become evident, and isostatic structure obscured.

On the other hand, the most commonly utilized scale-dependent approach to line generalization assumes that all features show unique progressions with scale change. Implementation of this approach most often results in maintenance of a separate feature data base for every intended scale of map representation. Many of the larger mapping agencies in North America currently take this approach. Problems with redundancy and errors in sheet matching and map comparison are of constant concern, not to mention the cost and sheer volume of disk storage required to archive all possible versions of every feature.

The converse issue to variation in versions of a cartographic line that differ in terms of their geometry is a more difficult aspect of generalization to address. Specifically, this is the question of whether a single simplification algorithm will affect different types of cartographic geometry in different ways. For example, the blocky appearance of coastlines produced by some computer mapping packages indicates oversimplification, and for very large coordinate files, the computations applied to simplify files beyond the usable limits of resolution reflect a loss of efficiency as well as reduction of visual quality. This is complicated by the fact that 'usable limits of resolution' will not be homogeneous within a coordinate file. For example, at very small scales, a simplification tolerance set to eliminate high frequency details along meandering streams may eliminate most of the meander pattern from very constrained ('youthful') channels while preserving older meanders and oxbow features quite accurately. At larger scales, approaching the 'limits of usable resolution', the older meanders may become simplified to the point of looking the same as a road, and not appear to be a naturally-occurring feature at all. It is useful to keep in mind that scale change may produce drastic shape transformations for some but not all features in a file. The effects of generalization will vary depending on the scale of the map and on the geomorphic nature of the features being simplified.

To preserve accuracy and recognizable details in simplifying a digital representation, then, the cartographer must be able to describe formally the details that must be preserved during the transformation. More specifically, the cartographer must be able to distinguish between those features that tend to vary geometrically with scale change, and those whose geometry remains invariant to changing scale. Without a consistent and rigorous definition of structural distinction, it remains difficult if not impossible for a single coordinate data base to provide multiple cartographic representations that are both accurate and recognizable. One possible solution is to mark digital files according to the geometry of the features they contain. This solution presumes that one may in fact determine different types of geometry for different types of cartographic features, and further, that unique structural progressions for each geometric type are apparent with changing scale. The remainder of this paper will propose one method for distinguishing between self-similarity and scale-dependence in cartographic features, and demonstrate its application to a variety of cartographic lines.

MEASURING THE LENGTH OF A CARTOGRAPHIC LINE

The goal of this analysis is to demonstrate the progression of one specific geometric characteristic of digital line files at a succession of finer and finer levels of graphic resolution. The particular geometric measure to be analyzed is line length, which is directly associated with the amount of fine details represented along the extent of a line. The length variable is also chosen for its direct relevance to the previous discussion of generalization as a transformation preserving maximum line length using a minimum number of coordinates. One of the earliest studies of line length progressions for various map resolutions was reported by Richardson (1961). He regressed the total line length against the spacing of map dividers used to step along the map feature, and found (Figure 2) that the length of geometric abstractions and regular polygons (such as the circle) stabilizes quickly, but the length of naturally occurring features (e.g., the coast of Britain) tends to increase as map divider spacing is decreased.

Two facets of this work are important to cartographic generalization. First is Richardson's concept of accuracy. he comments, "It is doubtful whether the total polygonal length of a seacoast tends to any limit as the [spacing of the map dividers] tends to zero." (Richardson 1961, p. 170). This is not to imply that the most accurate length is derived from the most precise measurement, but rather that accurate measurement becomes undefinable at the limits of measurement precision. The decision as to which level of precision will render the highest degree of accuracy was left to Richardson's readers. He merely points to the fallacy of over-reliance on precision in cartometric tasks.

The second important facet of Richardson's research is the derivation of differing slopes for different line features, and his allusion to differences in the 'characteristic irregularities' of cartographic lines. European researchers also studied map-measured distances (e.g., Steinhaus 1964; Perkal 1966; Srnka 1970). In these studies, too, equations were formulated so that 'actual' geographic length could be predicted based on the precision of measurement. All of the equations included a term variously identified as a measure of line character, irregularity, or sinuosity, and the term was most often described as a by-product

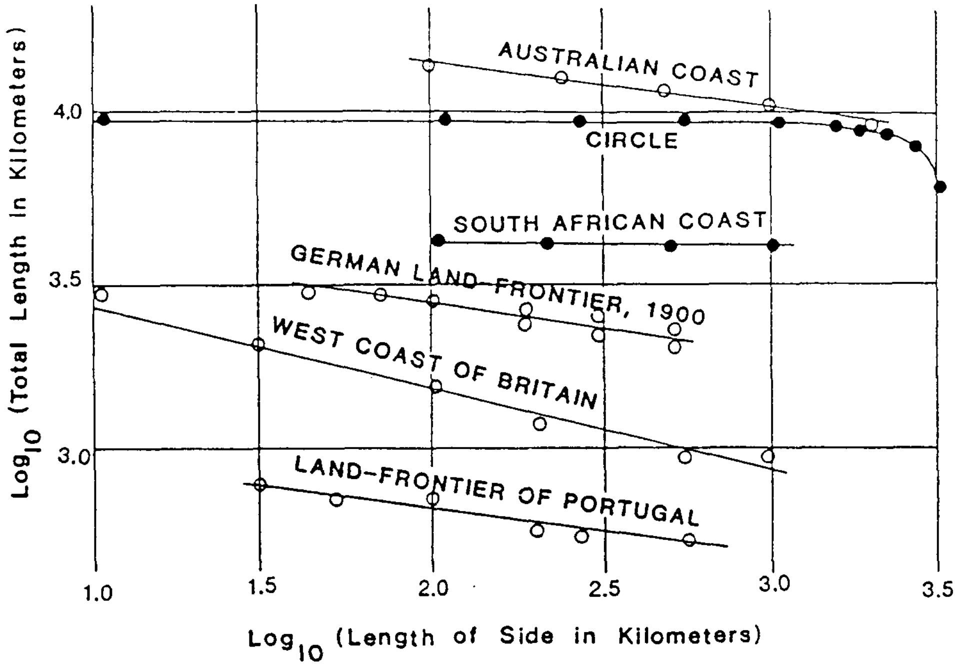

FIGURE 2 Richardson's empirical data on the rate of increasing length of linear features shows that for finer levels of resolution, length of naturally occurring lines appears to increase steadily while the length of a derived figure such as a circle stabilizes quickly. (After Richardson, 1961)

of the derivation. In Richardson plots, this term is the slope of the regression line, which varies from one line to the next. Richardson considered this variation a trivial finding: his interest was focused on the accuracy of cartographic measures. It remained for Mandelbrot (1967) to apply Richardson's slope value directly in his derivation of fractal dimension ($D = 1.0 -$ slope value) and to interpret the varying slope values in the context of structural complexity.

In this study, Richardson plots will be constructed for line features collected from digital data bases and by digitizing lines from maps, at several different scales. The plot construction will involve relaxation of two of Richardson's assumptions. The first assumption Richardson applied is that map detail occurs at equally spaced intervals, and this was incorporated into his measurements by utilizing equal spacing of map dividers along the extent of the map lines that he measured. Allowing map divider spacing to vary along the length of the cartographic features will be shown to provide a digital approximation of the line containing more original coordinate information and preserving more visual cues required for recognizability, without sacrificing the progression of line-length values. Relaxation of the assumption of a linear relationship between line length and map resolution will allow ready distinction between features whose structure is self-similar or scale-dependent. At those scales where a feature is self-similar, the Richardson plot will be linear; cusps of directional change will indicate structural changes that are scale dependent.

The application to generalization is the converse of Richardson's observation.

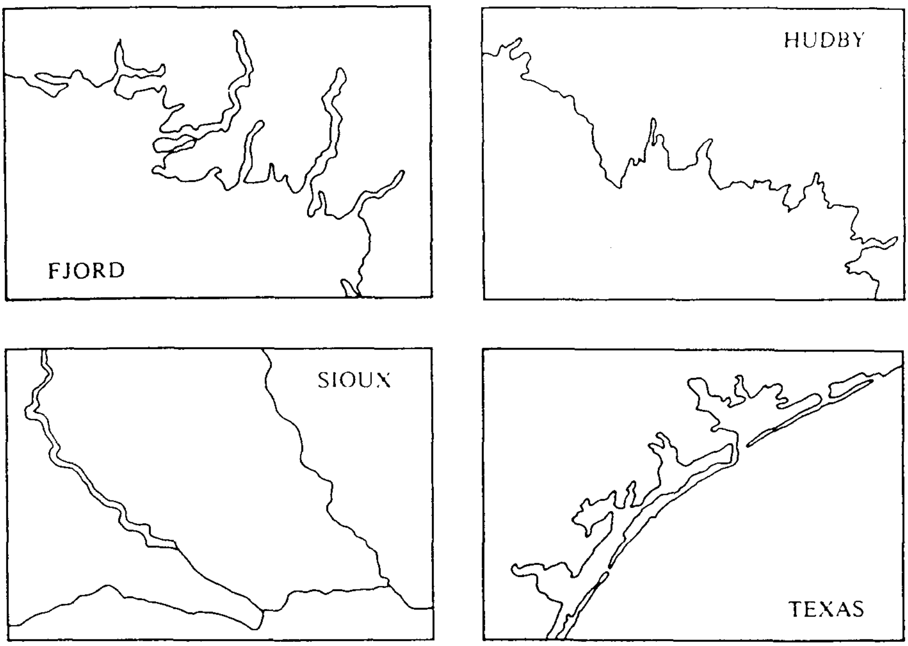

FIGURE 3 Four sample cartographic lines, collected from World Data Bank II, and originally digitized at 1:1,000,000.

He focused on the increase in line length as smaller details are resolved. In line simplification, the cartographer is concerned with preservation of line position (accuracy) and overall angularity (recognizability) as larger and larger details are eliminated. For self-similar features, where structure is replicated in proportion to resolution of measure, reduction of detail will be quite robust. That is, small changes in tolerance thresholds will not produce substantially different versions of a simplified line. At the cusp, however, line details will become sensitive to small changes in the simplification tolerance, and their elimination may produce substantial changes in the simplified line. It will be shown that cusps occur at unique levels of resolution for different features, that the Richardson plots can be used to identify them, and that knowledge of scale dependence can assist the cartographer in selection of simplification tolerance values.

LINE FEATURE DATA

Data for this project have been collected from a variety of map sources and map scales. First is a set of four lines culled from the World Data Bank II, at a scale of 1:1,000,000 (Figure 3). The lines are geomorphically distinct, and represent a submergent fjorded coast in British Columbia, a section of the Hudson Bay, (that is, an emergent coast), a depositional coast near Brownsville, Texas, and a dendritic river channel near Sioux Falls, South Dakota. Cartographic analysis of these line features has been reported elsewhere (Buttenfield 1986; Buttenfield 1987; Carstensen 1988). Also analyzed is a subset of the line files originally used in a study comparing simplification algorithms (McMaster 1987a; McMaster

1987b). These files were originally digitized from USGS topographic sheets at scales of 1:62,500 and 1:125,000, and were selected for topographic and geometric variety (McMaster 1983). Finally, two roads and a river were digitized from 1:50,000 scale maps (D. M. Mark, personal communication). These lines will be identified and displayed with their Richardson plots throughout the discussion that follows.

CONSTRUCTING THE RICHARDSON PLOTS

Relaxing the 'Map Divider' Assumption

Interpretation of Richardson plots is straightforward in most respects. Stepping along a length of coastline in equal increments set by mechanical map dividers may be simulated on a computer by approximating a coordinate string by straight line chords of a given length. The progression of smaller map-divider spacing requires reducing the chord length (Shelberg, Moellering and Lam 1982). Aside from the obvious accuracy lost by straight line approximations of a feature that may or may not include any of the original coordinates, the equal-step approach also ignores perceptual constraints underlying recognizability. That is to say, map readers don't necessarily recognize linear features in equal-sized pieces. A strong thread of psychological and cartographic research lends credence to this argument.

Attneave (1954) showed that visual cues in an image are concentrated along perceptual contours, that is, the edges of regions where a change of color, tone, or texture is perceived. Along the contours, information is concentrated at locations of maximum angular change. Kelley (1977) identified similar results in a cartographic domain. Kelley computed angular change along a series of coastlines and found significant correlation between the angular extrema and locations selected by subjects as being critical to recognition of the feature after simplification. His results imply that straight-line approximations by equal chords will preserve recognizability best when the directional change of the original line occurs in equal intervals, and that relaxation of the equal step constraint may provide more recognizable approximations for more complicated and non-regular features, such as naturally-occurring cartographic lines.

Marino (1979) used a similar research design to Kelley's, asking subjects to identify critical points along cartographic lines, but she did not analyze angular change. Instead, she repeated her experiment several times, insisting each time that subjects identify a smaller number of critical points. Her findings indicate that the coordinate subset selected for the most simplified representation will appear in subsets selected for more detailed representations, and that cartographic training does not appear to influence the hierarchical selection of points. This very important study justifies the presumption that a single detailed coordinate file may be used to represent maps at multiple scales.

These experiments provide a basis for understanding visual criteria used in cartographic recognition, but do not suggest how these criteria may be implemented in generalization tasks. Poiker's bandwidth approximation of a cartographic line (Peucker 1975) was formulated pursuant to a line-reduction algorithm (Douglas and Peucker 1973), whose computational efficiency resulted in widespread acceptance for computer simplification. Additionally, the subset of points selected at any level of simplification are by definition part of the original coordinate string, assuring some preservation of accuracy. And as White (1985) demonstrated, the points selected by this algorithm are very similar to the set of points that are identified manually as being critical for recognition.

It is possible to relax the equal-step constraint in producing Richardson plots by adopting the Douglas-Peucker line-reduction algorithm. Repeated iterations will subdivide the original coordinate file into two, four, eight, etc. pieces, with bifurcations defined at the location of maximum perpendicular distance from a straightline approximation. Straight lines (called anchor lines) connecting bifurcation points provide the straightline approximation, and anchor-line length may vary substantially along the length of the line. Ballard's (1981) strip trees produce an efficient data structure for archiving this information (Buttenfield 1986). Simplified versions of the line may be plotted by connecting the anchor lines across any level of the strip tree (Figure 4), to represent finer levels of resolution. The computations for the Richardson plot regress the logarithm of the total length of each anchor-line approximation against the average length of the anchor lines within that approximation. Table 1 shows length values for the 1:1,000,000 line features.

Richardson plots for these lines (Figure 5) demonstrate clear slope variation from one line feature to the next. The very flat slope of SIOUX and the very steep slope of TEXAS reflect differences in the size (amplitude) and density (frequency) of details. As the average anchor line length decreases, smaller-scale details are resolved for the TEXAS file; the SIOUX depiction does not contain much detail at finer levels of resolution, and its line length does not increase as quickly.

The slope variation is coincident with the tabulated data in Table 2, showing fractal dimension values computed from the slope of the Richardson plots. It is interesting to note that the fractal dimension computed for TEXAS by the Richardson method is greater than 2.00, contradicting Mandelbrot's fractional Brownian model. The nearly parallel slopes of FJORD and HUDBY indicate that these lines increase in length at an almost equivalent rate, and that their fractal dimensions are nearly equivalent, within this particular range of resolution (The column of significance values will be discussed in the next section.) As shown by this table, relaxation of the equal-step constraint does not preclude computing a fractal dimension; values have been computed directly from the slope of the Richardson plot, according to Mandelbrot's original derivation (Mandelbrot 1967, p. 637).

Comparison of Richardson plots for stream mode and point mode digitizing of the tamalp (a 1:62,500 wave eroded coastal feature) indicates that the rate of length increase does not change substantially as a result of encoding strategy, although the stream mode digitization is somewhat longer at every level of resolution (Figure 6). This should not be interpreted as being a more accurate representation, nor a more recognizable one. The similarity of slope values and of fractal dimension values indicate that for this feature at least, the amount of detail encoded point by point and in a time-dependent stream do not appear to differ greatly. The high r^2 values imply that a linear model is appropriate for this line, and that within this range of resolution, the feature is self-similar.

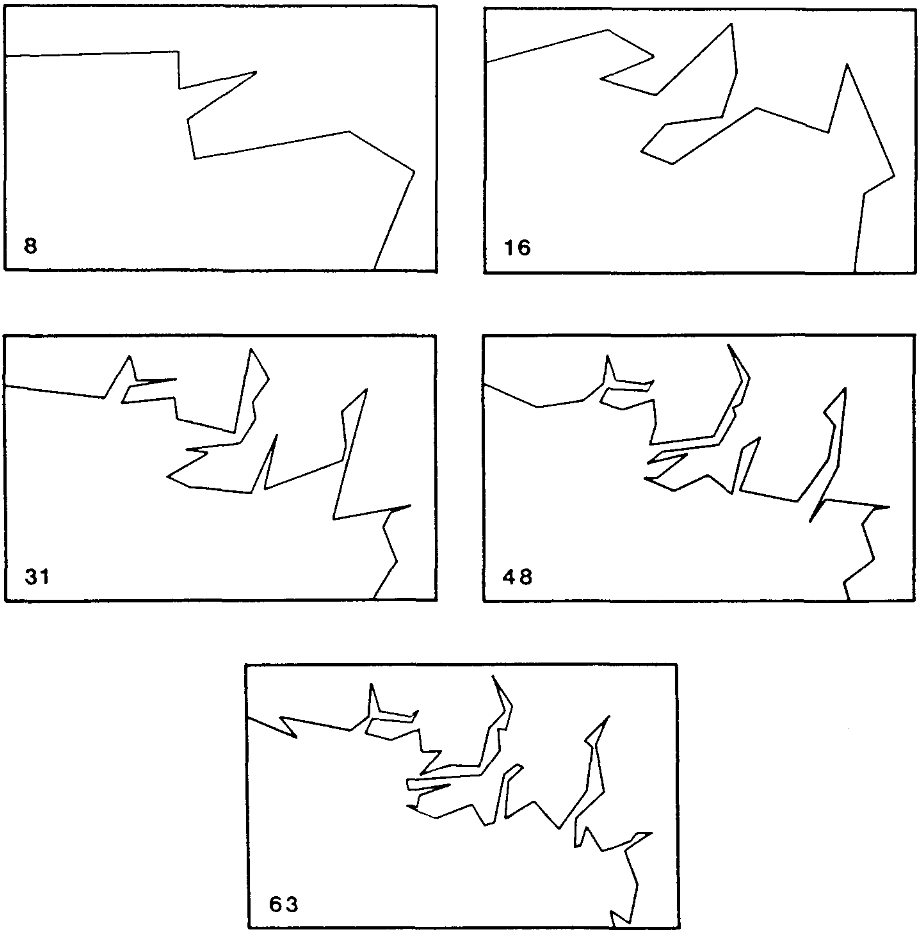

FIGURE 4 Anchor line approximations for the FJORD line. Each representation incorporates the anchor lines resulting from successive iterations of the line reduction algorithm reported by Douglas and Peucker (1973). Numbers indicate how many anchor lines appear in each iteration. Had smaller line details been resolved evenly along the line at every iteration, the number of anchor lines would progress by doubling (8, 16, 32, 64, and 128).

TABLE I RATE OF INCREASE IN ANCHOR LINE LENGTHS FOR 1:1,000,000 LINES
(values are logarithmic)

SIOUX		HUDBY		FJORD		TEXAS	
Average	Total	Average	Total	Average	Total	Average	Total
4.67	6.74	3.78	5.86	4.10	6.18	4.09	6.17
4.02	6.79	3.37	6.15	3.83	6.60	3.85	6.62
3.61	6.83	3.11	6.36	3.46	6.87	3.64	6.97
3.33	6.85	2.90	6.46	3.13	7.00	3.36	7.05
3.20	6.86	2.77	6.50	2.96	7.09	3.19	7.12

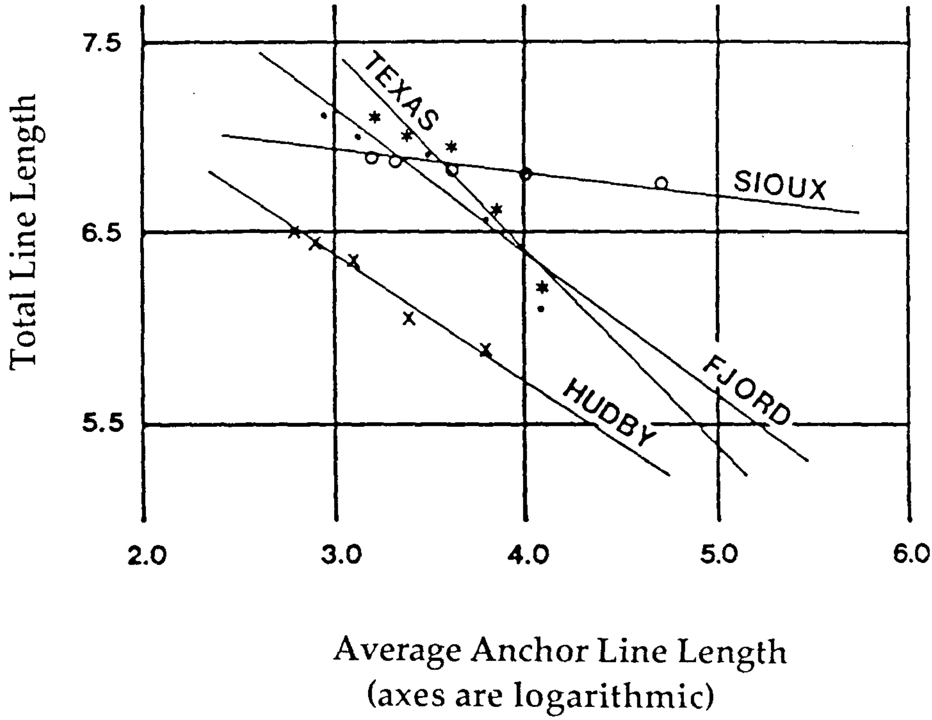

FIGURE 5 Linear Richardson plots for the four World Data Bank lines in Figure 3.

TABLE 2 RESULTS OF LINEAR REGRESSION 1:1,000,000 LINES

Line	r^2	Slope	Significance	Fractal Dimension
SIOUX	.99	−0.0826	.00002	1.08
HUDBY	.99	−0.6527	.0007	1.65
FJORD	.93	−0.7481	.0077	1.75
TEXAS	.88	−1.0185	.0188	2.02

Relaxing the Assumption of Linearity

Richardson plots for cartographic lines do not always display simple linearity. Richardson accepted this possibility implicitly, incorporating a nonlinear progression to represent the stabilization of the length measurement of a circle. He implied that similar equilibria would be reached in measuring the length of any regular polygon or derived (as opposed to naturally-occurring) figure. Richardson plots for cultural features do in fact display a similar curvature (Figure 7). In the Figure, no regression model has been applied; instead, observations have simply been connected, in sequence.

The two road features, both digitized from 1:50,000 maps (David M. Mark, personal communication) reach an equilibrium length in similar fashion to Richardson's circle. As discussed previously, the length of the line feature will

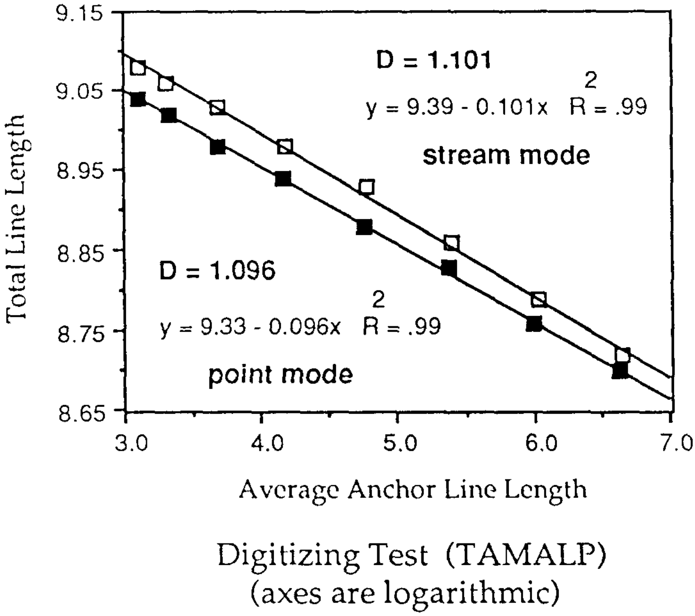

FIGURE 6 A comparison of Richardson plots for digitization of a feature in stream mode and in point mode. The line feature (TAMALP) is from the McMaster (1983) data, and is illustrated in Figure 9.

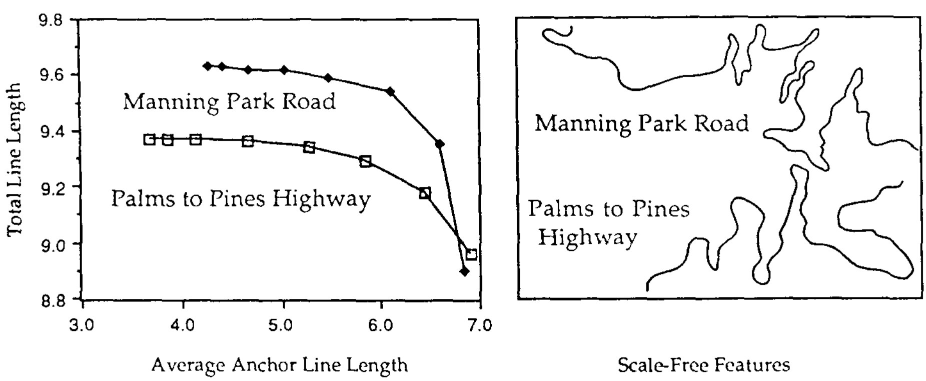

FIGURE 7 An example of cultural line features displaying characteristics of scale-free geometry. Compare these curves with the Richardson plot for a circle as shown in Figure 2. The two files were provided by David M. Mark. (Axes are scaled logarithmically.)

increase with finer resolutions of measurement, up to a finite limit. The constant radius of curvature and constant width limit the magnitude and frequency of angular change for roads, and this is reflected in their digital encoding on maps. Railroads, canals, fences, political and cadastral boundaries should have similarly shaped Richardson plots, due to similar geometric constraints, although this has not been verified empirically. In fact, one would expect that at the limits of usable resolution, any coordinate string should exhibit a similar equilibrium of line length. Storing Richardson plots (in nonlinear form) along with digital feature information may indicate immediately whether the digital version can be used to generate a reasonable graphic depiction at a desired scale, without having to wait to see the results of a simplification procedure.

For naturally occurring lines, the linear regression model has come to be accepted as the norm, and deviations from linearity are viewed in the same context as sampling errors falling randomly about the linear progression. It is an easy model to accept, because r^2 values are customarily very high (.99 is not uncommon) and are often associated with very high significance levels (see Table 2). The linear progression defines a constant rate of length increase for constant decreases in average anchor line lengths. It is a variation on the theme of statistical self-similarity, which some researchers (most notably Mandelbrot 1982; but see also Shelberg, Moellering, and Lam 1982) argue is an appropriate model for describing any type of structure whose complexity increases with resolution.

Anderle and Abrahams (1989) applied linear and nonlinear models to surface roughness measures on talus slopes, and found significant improvements in r^2 values for nonlinear models. Their work also indicates that the value computed as a fractal dimension varies (often substantially) depending upon the scale at which the measurement is computed. Similar results have been reported in cartographic research (Carstensen 1988). One should interpret this to mean that a single model of geometry is insufficient to accommodate the complexity of geographic features, and that digital descriptions of feature geometry should accommodate structures that appear self-similar for some ranges of scale, then change with sudden scale-dependence and display a new rate of increasing complexity for adjacent scales.

In this generalized model, scale-dependence is the expected type of geometric structure and self-similarity becomes a special case. By relaxing the assumption of linearity, such a generalized model is possible. This approach also avoids the fallacy of deciding *a priori* which nonlinear model (polynomial, trigonometric, 2nd order, 3rd order, etc.) to apply. The fallacy of course is that inflection points in the model will constrain where the cusps of scale-dependence will appear. If it is true that all geographic features are self-similar, then Richardson plots should retain their linear shape even when the linearity assumption is relaxed. But this is not the case. Compound linear patterns as shown in Figure 8 are much more common, and have been suggested in previous research (Mark and Aronson 1984). One might fit a nonlinear regression model to these data, but simply connecting the observations in sequence provides a good deal of information about the progression of line length. For self-similar structures, linearity will still be apparent, and sudden changes in slope will indicate levels of resolution at which the geometric structure is changing with changes in scale.

Both coastline features in Figure 8 are gleaned from McMaster's data, and were digitized from 1:62,500 topographic maps. The right part of the FAIRVIEW plot shows the loss of line length is steady when gross details are eliminated, and this is also evident in looking at the plot of the line feature. At the fourth level of resolution, the sudden change in the slope indicates a change in the rate at which line length is increasing. The steeper slope indicates a greater density of middle frequency detail; line length increases more rapidly as these details are resolved.

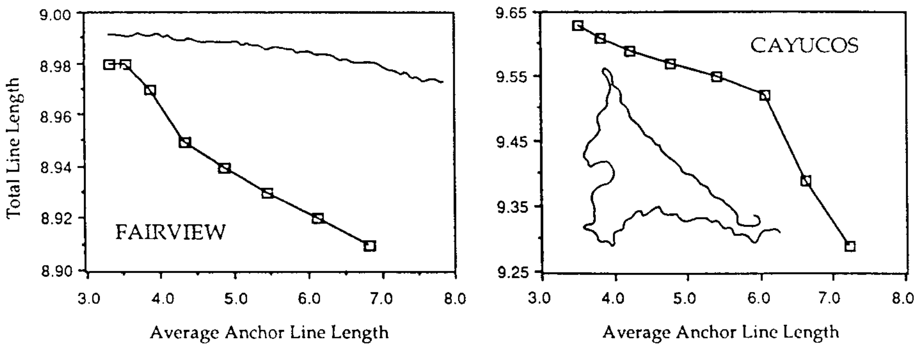

FIGURE 8 Compound linear patterns are often apparent in Richardson plots when the assumption of a simple linear relationship between resolution and line length is relaxed. FAIRVIEW illustrates a simple coastline along Lake Erie, and CAYUCOS displays a shoreline on a California coastal bay. The value 4.35 in Equation (1) is taken directly from the x axis of the FAIRVIEW plot. Both features are from McMaster's (1983) data, and were digitized at a scale of 1:62,500. (Axes are scaled logarithmically.)

On either side of this point, the plot displays two distinct slopes, and therefore two distinct fractal dimensions, separated by a cusp of scale dependence for details approximately 125 meters across. The cusp can be identified directly from the x axis of the plot (for this particular cusp, $x = 4.35$), given the scale at which data were originally digitized (1:62,500) and the units of measure for the original cartographic line. For McMaster's data, all coordinates are reported in digitizer units (1000 / map inch). The actual computation takes the standard form of scale conversion:

$$e^{4.35} / 1000 = 0.08 \text{ in.} = .20 \text{ cm} * 62500 = 125 \text{ meters} \qquad (1)$$

What does this imply for line simplification? Specifically, the details along FAIRVIEW greater than 0.20 cm in size appear to be self-similar, and line length increases in constant proportion within equal increments of scale change. This is indicated by the linear slope in the right half of the plot; at very coarse levels of resolution, simplified versions of the line should appear to change very little regardless of slight changes in the magnitude of the tolerance value that is applied. The steeper slope of the plot to the left of 0.20 cm indicates that the point $x = 4.35$ identifies a cusp of scale dependence. Tolerance values slightly below 0.20 cm will eliminate more details than tolerance values slightly greater than 0.20 cm. One might say that the structure of this feature is particularly sensitive at this level of resolution.

Applying computations from equation (1) above, a second cusp occurs in the plot at a resolution of 0.08 cm (roughly 50 meters on the ground). Between 0.08 cm and 0.20 cm, self-similarity is evident once again, although the rate of increasing line length (and thus the fractal dimension) differs. At resolutions below 0.08 cm, the horizontal slope implies that line length has reached an upper limit, and marks a limit to computational efficiency as well. That is, a tolerance value less than 0.08 cm will not elminiate any more meaningful details along the line. This may mark the limit of usable resolution for the digital file, although it is

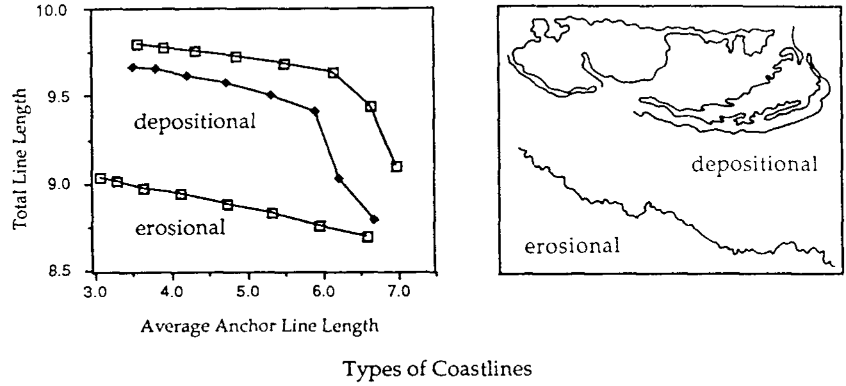

FIGURE 9 Distinctions between erosional and depositional coastlines may be evident on the Richardson plots, as shown here. The 1:62,500 features are part of the McMaster (1983) data set, and show MARTHAS and PROVTOWN, two wave-deposited coastlines on Cape Cod, Massachusetts, as well as TAMALP, a wave eroded coastline (also shown in Figure 6). (Axes are scaled logarithmically.)

difficult to confirm without extending the Richardson plot beyond the displayed levels of resolution.

CAYUCOS displays a different progression of line length, with increases that are dramatic at coarser levels of resolution (on the right) but tail off quickly below a resolution of 1.08 cm, taken from the *x* axis of the plot. In looking at the feature itself, the trend (gross detail) of the line doubles back upon itself, while the finer details are less sinuous and appear to be consistent along the extent of the feature. The cusp at 1.08 cm and subsequent stabilization of line length might indicate a good minimum tolerance value for simplifying this particular feature, as small details are not resolved as quickly as are the gross details. The cusp seems a point of diminishing return for further computations. Once again, a pattern of multiple progressions of self-similarity broken by cusps of scale-dependence is apparent, and the cartographer should attend to the sensitivity of details at the 1.08 cm cusp. The relaxation of linearity in the model does not preclude the existence of self-similar geometry, but rather accommodates it in addition to pointing out specific map scales where feature geometry may be most sensitive to slight changes in scale.

INTERPRETATIONS OF THE RICHARDSON PLOTS

What the nonlinear model of line length provides in addition to its cartographic implications is important for several reasons. The assumption of scale dependence has a certain geomorphical logic, as shown in Figure 9, Crude distinctions between depositional and erosional features are apparent in this triad of coastal features. It should not be concluded from this that all erosional features are self-similar, and all depositional features display scale-dependence. The more appropriate interpretation stems from the geomorphic differences in erosion and deposition along a coastline. Wave action along a coast will create differing forces of erosion at the headlands, and deposition in the bays (Strahler 1969). The differing results of these forces are apparent in the distinct details of the Richardson plots. It is important to realize that the geomorphic knowledge

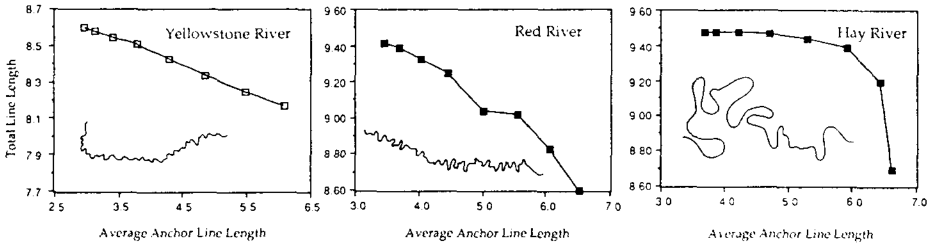

FIGURE 10 The three river features illustrate how linearity of the Richardson plot may be related to the degree to which meanders are constrained in their channels. The first feature shows a section of the Yellowstone River, described by McMaster (1983) as a section of a youthful stream flowing through a swamp, and the second (in the same data set) shows a portion of the Red River near Fargo, North Dakota. Both were digitized at 1:125,000. The third feature shows a portion of the Hay River in Canada, and was provided by David M. Mark. (Axes are scaled logarithmically.)

cannot be extracted directly from the Richardson plots, but rather that the plots can augment inferences and distinctions between stored digital information in a more efficient manner than plotting out every feature in the stored file.

This can be carried one step further, in cartographic inspection of the landscape beyond the coastline. One can understand that the adjacent elevation contours may remain unaffected by the geomorphic processes along the coast. One might anticipate that visible distinctions on a map between a coastline and its adjacent contour may increase with the age of the coast itself, and that nonlinear Richardson plots may provide a visual tool for identifying such distinctions. It would be interesting to examine the extent to which coastline age may be determined from differences evident in Richardson plots of the coast and its adjacent contours. This remains to be explored; but the scale-dependent assumption provides a means by which the research question may be posed.

Taking another example, one may move to a different conceptual level. Instead of distinguishing between naturally occurring and derived (contour) features, one may distinguish types of river meanders, or more specifically, focus on the progression of meander / channel widths with changing resolution. Figure 10 displays three river features. A section of the Hay River in Alberta displays very free meanders and a curvilinear Richardson plot, approaching a scale-free appearance. A section of the Red River (near Fargo, South Dakota) displays meanders of higher frequency and lower amplitude, and a Richardson plot approaching linearity. The Yellowstone River in Wyoming shows highly constrained meanders, and a channel that is nearly straight. The Richardson plot implies that within this range of resolution, this river section may be self-similar. One might interpret the degree to which meanders appear self-similar on the Richardson plots as a relationship between meander shape and number of half-meanders stored in the digital file. As before, relaxation of the assumption of self-similarity provides a visual means by which such research questions may be posed.

SUMMARY

It would appear that different types of line features display distinct progressions of length with changing map resolution. Continued research with digital files of

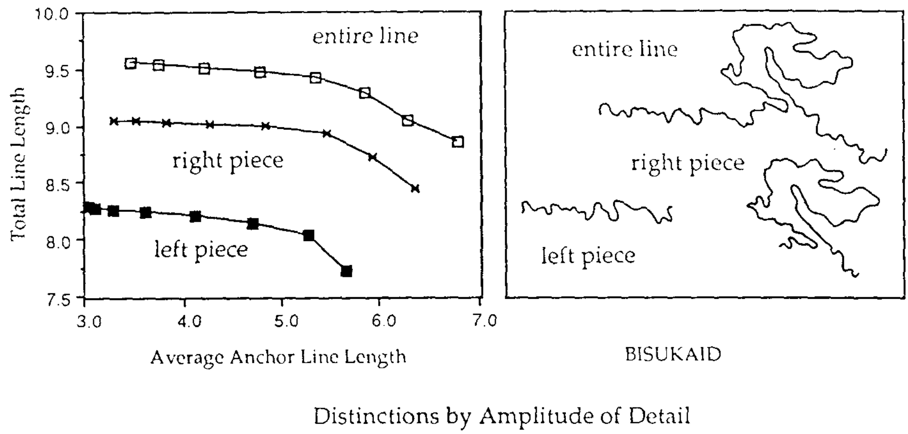

FIGURE 11 Distinctions of feature amplitude may be apparent on Richardson plots because of the relationship between feature amplitude and overall line length. The feature BISUKAID shows a contour on a plateau in Idaho, and was digitized (McMaster, 1983) at 1:125,000. (Axes are scaled logarithmically.)

different geomorphic features, across a larger range of resolutions, must be analyzed to confirm the questions raised in this research. The goal of such confirmation is to refine the flexibility of models applied in digital representation and map simplification to accommodate the complexities of the real world. Cartographers must evaluate methods for generalization and representation using comprehensive data sets, composed of features that are self-similar (or the special case of this, wherein features are scale-free), and features that are scale-dependent. If the goal of generalization is to preserve geometric structure, and structure can be seen to vary distinctly from one feature to the next, then it is possible that particular structures respond non-uniformly to different algorithms.

It should also be possible to use graphical tools such as the Richardson plots to detect automatically within a large coordinate file points at which the geometry changes, to modify tolerance values automatically, and to preserve visual character in a consistent and appropriate fashion. For example, one may use Richardson plots to distinguish between parts of a feature having differing amplitudes of detail (Figure 11), as in this 1:125,000 contour on an Idaho plateau. Although the Richardson plot for the aggregated line shows no clear cusp of scale-dependence, the feature displays clear differences in amplitude of detail for the subsections marked 'left' and 'right'. The Richardson subsection plots reflect this in two ways, by vertical separation of the plots, and by separation of a scale-dependent cusp. For the right piece, the cusp falls at a resolution of 0.60 cm (749 meters); and for the left, the cusp occurs at 0.49 cm (613 meters). The two subsections are sensitive to different tolerance thresholds, and an appropriate simplification tolerance for one part may render the other part blocky and unrecognizable. The Richardson plot comparison provides one way to decide where to modify the tolerance values, although not without the assistance of visual inspection.

On the other hand, some details may not be so apparent on the Richardson plot. Take for example the 1:125,000 contour from an arid plain, displayed in

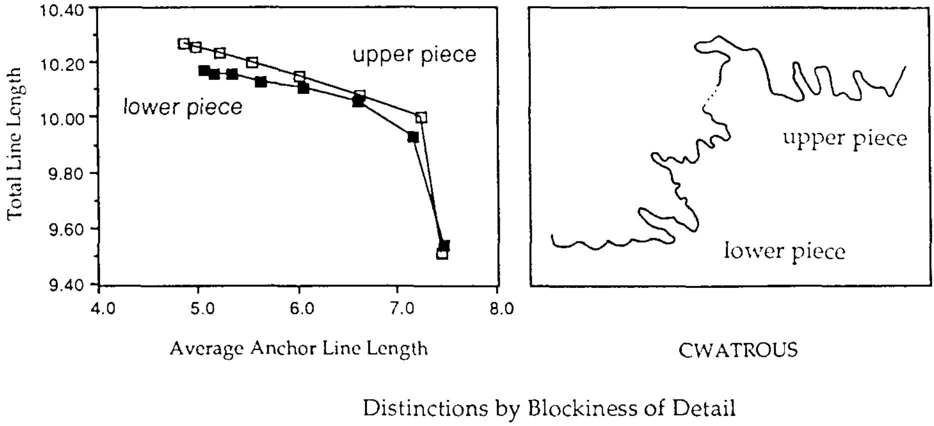

FIGURE 12 Distinctions of other geomorphic characteristics may not be visually evident in Richardson plots. Here, the two portions of the line feature CWATROUS show a contour on an arid plateau crossing two differing types of bedrock material. The feature was digitized by McMaster (1983) at 1:125,000. (Axes are scaled logarithmically.)

Figure 12. The contour appears to cross a rock type or soil boundary, beyond which water cuts through surface material more easily. The visual evidence for this is the change to sharply angular from more blocky and square-shaped indentations. On the Richardson plots, these differences are not apparent, and amplitude and frequency of linear detail appear to be quite similar, even though the visual character of the two parts of the feature is very different. Obviously, a single graphic such as the Richardson plot is not sufficient by itself to distinguish all kinds of structural detail. Other techniques have been suggested in the literature, for example Witkin's (1983, 1986) structural 'fingerprints' and Buttenfield's (1986) structure signatures. Demonstration of the use of either of these tools in conjunction with Richardson plots remains to be explored.

Selection of appropriate tolerance values during feature simplification must accommodate the changes in structure encountered along the extent of line features. Computational efficiency will be lost if the tolerance threshold is so small that all details are retained in the transformation. Conversely, application of tolerance values that are greater than some maximum threshold will eliminate even gross details, and may compromise recognizability. It is clear that line features display unique progressions of structural detail, and that cartographers must not establish and modify tolerance values with blind faith throughout a coordinate file. One may presume in the case of a self-similarity that since details are replicated at every scale, tolerance ranges may be chosen within a range of values without loss of important details. At the cusps of scale-dependence, however, even small fluctuations in tolerance thresholds may change the look of the cartographic product dramatically. The question of what tolerance ranges are appropriate to a given class of features remains to be determined, both numerically and perceptually.

Until cartographic researchers better understand the nature of the features represented on maps, the constraints of the models designed to represent or simplify those features in digital form will not be fully understood. Assumptions of self-similarity cannot accommodate geometric characteristics that change with the scale of graphic depiction. Furthermore, the algorithms and techniques applied during map generaliztion must preserve scale dependence if it is present, in order to allow multiple map representations to be produced from a single data base, and to direct automation of selection and modification of tolerance values. Finally, until researchers calibrate generalization against features of both self-similar and scale-dependent classes of geometry, results of cartographic research will remain domain-specific and therefore not readily comparable. This will impede feature simplification and terrain representation alike. In the rush to develop geographic information systems and to automate the mapping process, these questions should be included on the research agenda.

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ABSTRACT This paper provides a typology of two classes of geometry for cartographic lines, based on the well-known Richardson plots. The first class contains objects having self-similar geometry, features whose structural characteristics are replicated either precisely or statistically with changes in scale. Self-similar features are currently described by fractal models, which some argue are appropriate for all cartographic objects. The fallacy of this statement will be demonstrated. The second class of features is fully scale-dependent, and contains cartographic objects whose geometry varies distinctly with changing scale. Both models are described and applied to examples of digital line features, to demonstrate their worth in encoding and preserving particular types of cartographic detail during automatic generalization.

RÉSUMÉ Cet article présente une typologie de deux classes de géométrie de lignes cartographiques, à partir des réputés tracés de Richardson. La première classe contient des objets qui conservent une géométrie identique, des éléments dont les caractéristiques structurelles sont copiées, soit précisément, soit statistiquement, lors de changements d'échelle. Ces éléments ayant une géométrie constante sont couramment décrits au moyen des fractales qui, comme le soutiennent certains, conviennent comme modèle pour tous les objets cartographiques. La fausseté de cet énoncé sera démontrée. La deuxième classe d'éléments est pleinement dépendante de l'échelle et renferme des objets cartographiques dont la géométrie change clairement lors de changements d'échelle. Les deux modèles sont décrits et accompagnés d'éxemples d'éléments linéaires numériques, afin de démontrer leur valeur pour encoder et préserver certains types particuliers de détails cartographiques lors de la généralisation automatisée.

ZUSAMMENFASSUNG Der Artikel erörtert eine Typologie von zwei Arten Geometrie für kartographische Linien, die auf den bekannten 'Richardson Plots' beruht. die erste Art umfaβt Objekte mit selbstähnlicher Geometrie, also Formen, deren strukturelle Kennzeichen entweder genau oder statistisch mit der Maβstabsänderung kopiert werden. Selbstähnliche Elemente werden derzeit durch fraktale Modelle dargestellt, von denen mancher behauptet, sie paβten für alle kartographischen Objekte. Der Trugschluβ dieser Feststellung wird bewiesen. Die zweite Art von Elementen ist vollkommen maβstabsabhängig und enthält kartographische Objekte, deren Geometrie deutlich met der Masβstabsänderung wechselt. Beide Modelle werden beschrieben und auf Beispiele von digitalen Linienfiguren angewendet, um ihren Wert zum Erfassen und Erhalten von bestimmten Arten kartographischer Elemente während der rechnergestützten Generalisierung darzustellen.

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