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PRINCIPLES AND APPLICATIONS

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GENERALIZATION OF SPATIAL DATABASES

J-C MULLER

Generalization procedures to transform and model spatial databases for analytical or display purposes are notably absent within the realm of functions presently available in GIS. The many motivations for generalization include the need for economy, data robustness, multipurpose use, and display and communication. Generalization has an impact on data quality characterized by locational and attribute accuracy, consistency, and completeness. A number of concepts, procedures, and techniques have been developed for generalizing spatial databases and the most important raster and vector approaches are reviewed. A catastrophic approach to generalization is proposed, which shows the way objects undergo sudden rather than smooth changes in the way they are depicted. Finally, the merits of several holistic approaches to generalization are discussed.

INTRODUCTION

Generalization, in its epistemological sense, is a process which attempts to establish the universality of a statement (Hawkins 1983). Such meaning needs to be re-emphasized at a time when there seems to be a confusion between the operational aspects of the procedure and its ultimate goals. Generalization, of course, is about 'the selection and simplified representation of detail appropriate to the scale and/or purpose of a map' (ICA 1973). But this definition is misleading because it confuses the tools and the objectives. A map, and more generally a spatial database, is a statement about geographical reality and generalization is a process which tends to universalize the content of that statement. Whether generalization is viewed as a transformation operation (Tobler 1966), or as a modelling operation (Hake 1975), its goal is to establish what is universally of interest for the scientist. It is an information-oriented process. These transformation and modelling procedures are subject to a number of constraints, such as maximizing information content while observing specific restrictions (e.g. minimum scale, minimum geometrical dimension and minimum accuracy standards). Generalization is performed for map display and communication purposes, but also, and perhaps more importantly, for analytical purposes. The necessity to understand at which scales or range of scales spatial processes occur is one of the driving forces behind generalization today.

Attempts have been made recently to automate the procedures of generalization. This development, in turn, has led to efforts to formalize a process which had remained highly intuitive and subjective. The identification of rules and their implementation into a system which can simulate the work of a traditional cartographer is one of the most difficult challenges facing the GIS research agenda of the 1990s. Whether those efforts will be successful is still uncertain. Robinson, in the second edition of *Elements of Cartography* (1960: 132) already alluded to these difficulties: 'many cartographers have attempted to analyze the processes of generalization, but so far it has been impossible to set forth a consistent set of rules that will prescribe what should be done in each instanc It seems likely that cartographic generalization will remain forever an essentially creative process, and that it will escape the modern tendency towards standardization. . . .

This chapter reviews the procedural and technical issues of computer-assisted generalization, against the interlacing background of scale and spatial resolution. The motivations for generalization are considered first. Various concepts of space which correspond to various levels of generalization are envisaged. Database requirements and data models for generalization are then mentioned, with a focus on the vector and the raster models. In the section on procedural tools, a distinction is made between heuristic and rule-based solutions to computer-assisted generalization. The methods of vector and raster generalization are then outlined. This is followed by discussion of independent versus scale dependent databases and the idea of a comprehensive, holistic solution to the generalization of spatial databases. Finally, some brief conclusions are presented.

MOTIVATIONS

Generalization is not only motivated by a reduction of scale representation, as it sometimes appears to be in cartographic manuals (Swiss Society of Cartography 1987). Such a point of view overemphasizes the display and legibility constraints in map production. Generalization consists of the application of a transformation to spatial data and is prompted by four main requirements:

1. *Economic requirements*. Our knowledge of the universe is determined by our data collection procedures which are influenced by financial and technological constraints. The only source for which a database is made available is usually already generalized through discrete sampling procedures. Obvious examples are databases created from existing digitized maps, or data aggregated into census units. Another example is the generalization performed during the acquisition of original topographic information by a topographer or a photogrammetrist. Simplification, selection, geometrical and conceptual combinations are carried out which result in a reduction of information. This process, termed object generalization, produces a primary model of the real world, that is, a topographic basic map referred to as a 'Digital Landscape Model' (DLM) (Grunreich 1985).
2. *Data robustness requirements*. Errors in spatial databases occur at all stages, during data collection, data recording and data manipulation. The sources of error are human, instrumental as well as methodological (i.e. wrong classification procedures). It would be a fallacy to believe that an increased precision in measurement or more extensive data sampling would decrease the chances of error in interpretation. The reverse is probably true, because the true value of a single observation whose measurement is affected by random errors may be hidden through some high-frequency disturbance. Hence, we need generalization in order to filter out the errors and consolidate the trends. A generalized trend is more robust than an individual observation. Smoothing operations to generalize curves and surfaces make this assumption; for instance, there is the notion that if the observations had been more accurate the curve would have been smooth (Whittaker and Robinson 1944).
3. *Multipurpose requirements*. Official surveying and mapping organizations must provide up-to-date topographic–cartographic information for a variety of users (regional planners, geoscientists, ecologists, military, etc.). This information must also be provided at different scales, since natural or human features display scale-dependent properties 'and those levels at which scale-dependence becomes apparent vary from one feature to the next' (Buttenfield 1989: 81). Hence information must be filtered and modelled according to usage and scale significance. In order to adhere to the economic principle of singular acquisition and multiple use, a data flow from the original DLM to a lesser resolution or special purpose DLM must be established through model generalization (Grunreich 1985).
4. *Display and communication requirements*. This is probably the best known motivation for generalization. Decision making in the context of GIS relies heavily on the use of communication maps. These maps represent spatial information collected usually at larger scales through different means, including ground survey, analytical photogrammetry and satellite imagery. Generally, there is a need for data compaction or data compression since the

amount of data collected is much more than can be visually communicated. The notion that there are physical limits to the amount of information that can be displayed on a map is shared by both traditional cartographers and the GIS community. Some thresholds have been suggested (a maximum density of ten graphical marks per square centimetre), beyond which a map becomes illegible (Bertin 1967). The 'heretical alternative: plot everything, allow features to overlap or merge, producing blobs of clutter in high density areas, and then allow the user to zoom in to resolve the ambiguity' (Mark 1989: 69) is intellectually interesting, but requires a computer environment and would not allow the recognition of pattern generating processes at smaller scales. Furthermore, the scale reduction of objects and forms cannot be continued indefinitely. It should terminate when the limit of acuity of the human eye is reached (Swiss Society of Cartography 1987). Therefore, generalization must be used to select, simplify, exaggerate and symbolize information in order to afford communication and understanding. Cartographic symbolization of the digital landscape model leads to the 'Digital Cartographic Model' (DCM) as shown in Fig. 30.1.

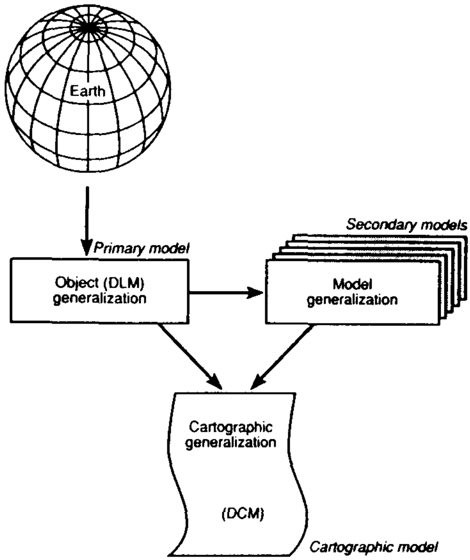


Fig. 30.1 Models derived from the primary Digital Landscape Model (DLM) are special purpose secondary models of reality. They are free of cartographic representational information. Both primary and secondary models may be used to create a cartographic representation of a Digital Cartographic Model (DCM) through the process of cartographic generalization (after Grunreich 1985).

GENERALIZATION IN DATA QUALITY

Data quality is usually characterized by four components:

1. Location accuracy
2. Attribute accuracy
3. Consistency
4. Completeness

Generalization of spatial databases may influence some of these components. Locational accuracy usually decreases through the process of generalization. Lower local accuracy may in turn affect attribute accuracy. As the locations of objects become more uncertain their attributive characterization becomes fuzzier and more complex. Generalization may also affect completeness, since the information is reduced to major trends (e.g. series of parcels may display an anomaly which disappears when looking at them as a community). Consistency could be affected by uneven applications of spatial or temporal abstractions, with inconsistent disappearance or emergence of scale- or time-dependent features. The potentially negative (and positive) effects of generalization on data quality will be considered in the broader framework of relationships between generalization, scale and accuracy, and resolution.

Generalization and scale

A fundamental issue is to decide at which scale the information should be generalized. Ideally, it would be useful to be able to vary the scale according to the level of precision required. Naturally-occurring features often require larger scale for their portrayal than cultural features. This raises intriguing problems of metric representation, but the idea of variable or elastic scaling within a single map is not new (Muller 1982). One might ignore the problem of representation and vary the level of precision (which amounts to a scale variation) at which spatial objects, such as highways and rivers, are encoded in the database. 'Asking the scale of a database is in fact making a query about the lineage of the database' (NCGIA 1989: 13). This lineage is not the same for all objects, some being better documented and requiring more codes of description than others.

Generalization and accuracy

Both statistical and cartographic generalization affect the accuracy of spatial databases. Statistical generalization is a filtering process whose aim is spatial modelling of attributive information attached to locations. Objective constraints, which are imposed during the process of statistical generalization, are the preservation of the spatial mean, variance and form of the distribution. A wrong classification, for instance, may hide the characteristic pattern of a statistical surface. Furthermore, a classification may create crisp boundaries between areas whose boundaries are naturally fuzzy.

Cartographic generalization, the aim of which is visualization, can affect locational accuracy to a great extent. Features may be displaced and their original shape may be distorted. Several criteria have been applied to estimate the accuracy of cartographic generalization, such as minimum vector displacement, minimum change in angularity, preservation of parametric characteristics, and self-similarity. Examples of parametric characteristics are the overall sinuosity of a line, the relationship between x,y coordinate point values and point distance from the origin, as well as the 'structure signatures' proposed by Buttenfield (1986). In contrast to angular deviations and areal displacements, which are derived from a cross-comparison between two lines, a parametric characteristic describes the intrinsic geometry of a single line. A different parameter which could be used is the relationship between line length and resolution of measurement. When the relationship is plotted linear on a log–log graph (power function), the line is said to be statistically self-similar (see Gatrell 1991 in this volume for further discussion). Ideally, the geometrical structure of self-similar objects should not be affected by generalization (Muller 1987). Another criterion is the preservation of relational accuracy. Whereas locational accuracy is partly lost, orientation and connectedness of objects in space must be saved. Quite often spatial databases are created from existing digitized maps. Somehow these digital versions are revered and there is a tendency to believe that they are more accurate than their analogue counterparts. Those databases are only a result of cartographic generalization, however, and are not as reliable as those products referred to earlier as Digital Landscape Models.

Generalization and resolution

Spatial resolution refers to the ability of a recording system in distinguishing closely spaced objects. Clearly, in the case of databases, spatial resolution is determined by the quality of the sampling which was conducted by the data collector (see Fisher 1991 in this volume). A fine sampling will provide better resolutions than a crude one. Furthermore, data may be generalized by resampling at coarser levels of resolution. Hence, generalization will tend to increase the size of the smallest detectable feature.

The way data are sampled is usually not homogeneous, and spatial resolution varies from place to place. The assumption here is that spatial objects and processes vary in size and have different wavelengths. Hence, the data must be reported using different spatial resolutions. For instance, triangulated irregular networks (Weibel and Heller 1991 in this volume) are usually more efficient than uniform sampling grids because they can adapt to the variable conditions of the terrain. Another example is the changing spatial resolution of the county network in the United States. The spatial subdivisions reflect the historic evolution of population settlement and density (Fig. 30.2). Resampling into bigger units, which would reduce or remove (such as through a square lattice) the variance of spatial resolution between east and west, would relatively over-resolve the data in some parts and under-resolve in others. Hence, generalization through coarser sampling must be applied by relative equal amounts taking into account the changing size of the original sampling network. In effect, this means preserving the variance of the original sampling resolution.

As generalization tends to reduce resolution,

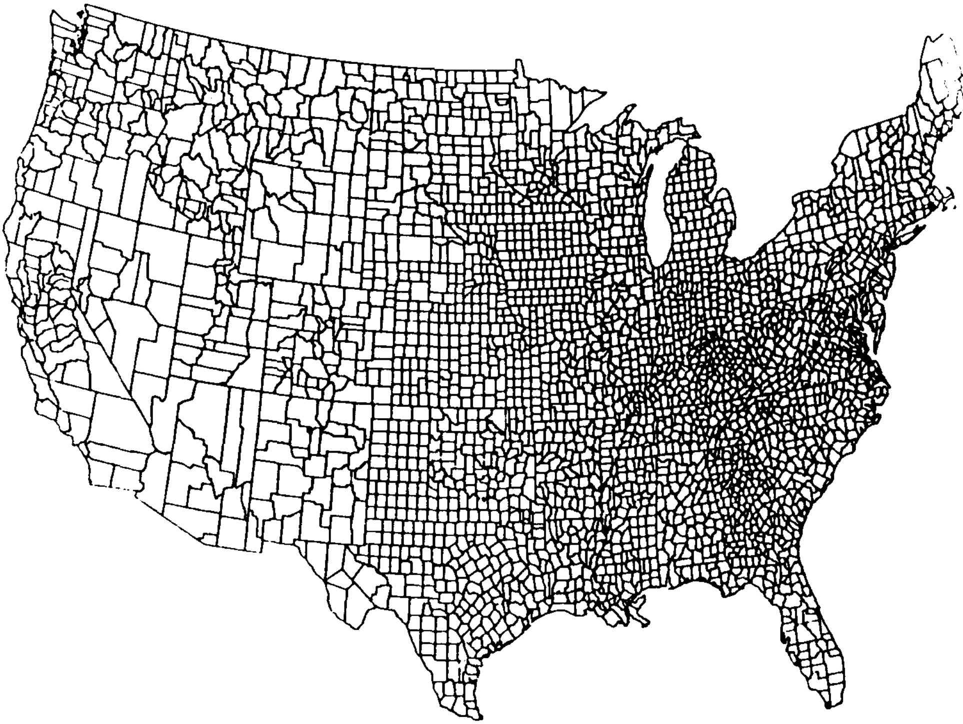


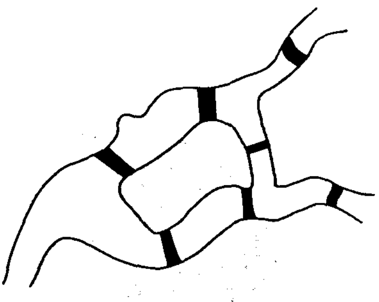
Fig. 30.2 *County map of the United States. Uneven sampling reflects uneven population densities as well as* historical differences (earlier settlement in the east).

some features which were apparent at larger scale disappear, while others which were previously concealed may emerge. Those are resolution-dependent or scale-dependent features and the purpose of generalization in this case is to uncover those underlying patterns which would otherwise remain hidden. A coarser resolution or a higher level of generalization may provide more explanation over the variance of a spatial variable than finer resolution levels (Tobler and Moellering 1972).

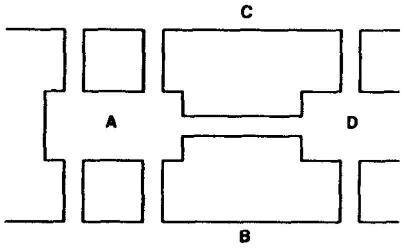
CONCEPTUAL MODELS

Conceptual models are used to provide a framework in which to express spatial relationships. There are basic categories which are recurrent in the GIS literature, such as vector space or raster space, and which reflect the form in which data are basically encoded, organized and manipulated (Egenhofer and Herring 1991 in this volume). We have become so entrenched in those categories that we now have raised the procedures and techniques of handling spatial data to the status of epistemological models. The basic distinction between vector and raster formats also has ramifications in the methods which are used to generalize spatial databases. Those methods will be reviewed in an independent section.

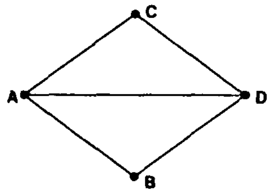
Other categories of space, which are of concern to spatial scientists and which represent different levels of generalization, are the metric, topological and graph-theoretic representations (Gatrell 1991 in this volume; see also Fig. 30.3). The metric space describes the distance relations between spatial objects, and constitutes the lowest level of abstraction. Due to the physical limitations of our tools to measure and represent the position of spatial objects, a discretization takes place which is part of generalization.



Metric representation



Topological representation



Graph-theoretic representation

Fig. 30.3 Three representations of Königsberg, USSR, corresponding to three different levels of generalization.

The topological space, instead, deals with the existence of connectedness relations between points in space. For instance, connections between areas, across boundaries or between settlements via a set of transport links may be described independently of distance. Hence, spatial databases can be derived which do not include any metric information. Topological relations may be encoded through tables (Figure 30.4).

A higher level of abstraction is reached through graph-theoretic representations. Here we are only concerned with the conceptual structure of spatial information depicted through graphs in which nodes express concepts and arcs denote relations (Sowa 1984). Neither the metrics nor the topology are preserved. A graph-theoretic mapping of market places (concept) and first-order links (relation) is an example of such representation.

Note that these categories of space illustrate various degrees of generalization at the conceptual level, and do not concern the problems of generalization at the display level. They are meant to provide a theoretical foundation for data structuring. Data structures are determined by requirements in accessibility and manipulation of information whose degree of generality may vary according to the type of GIS analysis which must be performed.

Database requirements

One basic requirement is that 'the database must support a wide variety of products covering various geographical extents across the globe' (Guptill 1989:439). The user must be able to access information at various levels of detail, from local to global scales, and at various levels of abstraction, from individual objects to the 'envelope' of their classes. In other words, geometrical, topological and thematic relationships between 'real world objects' must be stored or be derivable computationally (Rhind 1988) for geometrical description as well as for hierarchical classification and thematic retrieval of spatial entities.

Another requirement is that a database must be based on spatial proximity at both the metric and topological levels. This characteristic is fundamental for accessing local information, identifying neighbours or performing cartographic generalization.

Finally, the database must be object oriented to afford an object-oriented programming approach to generalization. In an object-oriented environment, procedures are bound to the object itself, that is, objects are active and execute their methods (i.e. generalization) directly in response to a received message. Messages usually consist of a destination (i.e. shoreline), an operator name (i.e.

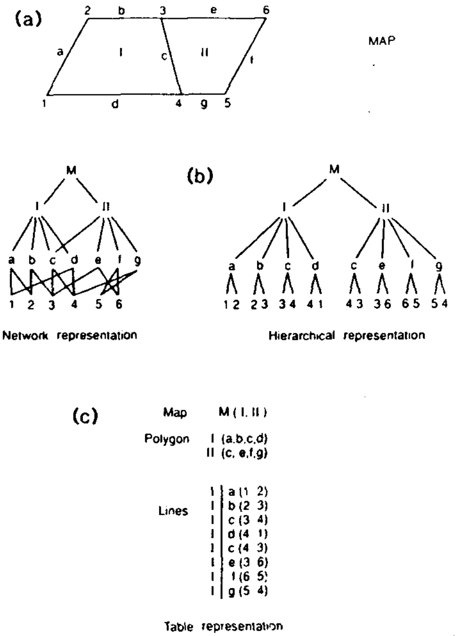


Fig. 30.4 Various ways of representing topological relations: (a) cartographic form; (b) network and hierarchical diagrams; (c) table.

simplification), and a parameter (i.e. bandwidth tolerance). In an object-oriented system, all objects belong to classes. In turn, each class of objects defines instance variables (location, size, colour) which must be instantiated when an individual member of that class (say, a house in the class residential area) is created. Finally, each class is one descendant of a more general class (settlement) that defines the common properties of all its children. Object-oriented programming has gained much popularity in the area of computer graphics. Its main advantage seems to be the ability to support the abstraction of both data and procedures, afford the interaction between objects, and ensure consistency through inheritance trees (Luger and Stubblefield 1989). For instance, the various representations, at different scales, of a single feature may be linked so that they 'inherit' common characteristics. Any update on one of the representations could then appear concurrently across all map-scale layers in the database (Mark 1989). Much work will have to be done, however, in defining what the semantic of a truly object-oriented approach for automated generalization should look like.

Database models

The data models holding the best promises for automated generalization have been based on hierarchical concepts. Strip or binary tree representations (Samet 1984), where a curve is recursively subdivided until a predetermined value of the strip width is reached, lead naturally to the automated generalization of lines.

Hierarchical tessellations of space, whether in the form of triangulation, square or hexagonal tiles, also hold promise. Where space is modelled by quadtrees areas may be visited using an *N*-shaped path based on the Peano curve (Peano 1890; Morton 1966). Using Peano relations, spatial queries can be resolved elegantly and quickly using Peano tuple algebra according to the rule: a quadrant (--> a tuple) can always be split into four quadrants (tuples). This may be expressed in the form: PR (node#, Peanokey, size). Here, PR denotes a Peano relation; node#, the region name or number; Peanokey, the Peano key of the quadrant; size, the size of the quadrant. This kind of locational relationship may be advantageous for generalization operations, since spatial resolution (determined by the quadrant size), is not a constant but varies according to the density of spatial information. Highly dense information leads to the formation of small quadrants, whereas large quadrants indicate sparse information. Hence, a spatial query of all tuples of small size would yield all areas where more generalization is required. The geodesic hierarchy, referred to as the Quaternary Triangulation Mesh (QTM), is another structure which lends itself to automated generalizations: 'Larger tiles (shorter geocodes) have fuzzier locations than smaller ones (longer codes), and thus have less positional certainty. This property can help one distinguish boulders and fenceposts as monuments' (Dutton 1989:48).

There is a need for comparative research to evaluate the performance of database models for automated generalization.

PROCEDURAL TOOLS

GIS which are presently available on the market offer few capabilities for database generalization. Generalization functions are mostly limited to line thinning, dissolving lines and merging attributes, as well as smoothing of digital terrain models. Those functions are usually *ad hoc* and are not supported by a theoretical foundation. For instance, decisions based on spatial autocorrelation are rarely applied in the process of merging polygons. Aggregation to higher administrative units is mainly achieved on the basis of a statistical classification, without considering neighbourliness relationships. Herzog's generalization method based on adaptive filtering processes is a step in the right direction (Herzog 1988). The method assumes spatial autocorrelation and takes into account the value of a polygon weighted by the values of its neighbours as well as the length of their common boundaries in order to reach a decision on aggregation. Generalization has defied GIS designers because contrary to other GIS functions, such as polygon overlay or edge matching, the procedures are not well defined and are difficult to formalize.

Manual versus automated generalization

There is much to be learned from manual methods in the geographical and cartographic disciplines, since statistical and cartographic generalization have been used successfully for many years by human experts. Statistical generalization is spatial modelling for the purposes of spatial analysis, whereas cartographic generalization is performed for the purpose of visual communication (Brassel and Weibel 1988). The first type of activity may also be prompted by the needs of communication, but is usually undertaken for other purposes, such as the extraction of a subset of an original data set for data analysis. The second type of activity, on the other hand, is *always* used for graphic display. There are three kinds of knowledge which are required for generalization:

1. *Geometrical*, where size, form, distance and connectedness are assessed.
2. *Structural*, where the underlying generating processes which gave rise to a phenomenon are analysed.
3. *Procedural*, where appropriate tools (including simplification, selection, classification and symbolization) are identified.

Objective and subjective thinking are both involved in manual generalization. They do not play an equal role, however, and it is obvious that statistical generalization provides a better basis for objective thinking than cartographic generalization. Statistical generalization intervenes mainly in the thematic (attribute) domain and automated solutions based on classification or seriation have already been developed (Muller and Honsaker 1983). One example of techniques applied in the temporal domain is time series analysis. Cartographic generalization, on the other hand, is a conglomerate of many different processes which are difficult to bring into one unified, formalized theory. This is particularly true for those kinds of transformations which are traditionally dependent on visual judgement, such as symbolization. Most efforts towards automation have concentrated on the geometrical aspects of cartographic generalization using various types of procedures. Recent research advocates a rule-based approach to handle the structural and conceptual aspects of the generalization process.

The procedural approach

The procedural approach uses algorithms for the execution of numerical operations. Algorithms have rules expressed in the form of 'IF THEN' statements. They are called conditional statements and are based on string or numeric matching. To arrive at a solution, one must execute those statements in an order which is predetermined by the logic of the program.

The algorithmic or heuristic approach to generalization is typically a very specialized, narrow solution to a specific problem. The thinning of geographical lines through selection and simplification algorithms is a well-known example. Numerous algorithms to perform this task have been published and continue to be published (Zycor 1984; Li 1988; Thapa 1988). Most notable techniques involve the use of epsilon filtering (Perkal 1965) and bandwidth encoding (Peucker 1975). They are based on the view that a digital line is an ordered set of two or more coordinate points. An improvement to this rather simplistic approach is a generalization based on parameterization and self-similarity of the line which attempts to retain its global parametric constituents. The parametric approach was prompted by the idea that the geometrical structure of a geographical line is representative of a physical or human process, and that this structure must be preserved. The Psi–*s* plot, for instance, has been used by O'Neill and Mark (1987) to differentiate river meanders. The Psi–*s* plot is a single valued function where the orientation angles at points along the line are plotted against *s*, the cumulative curvilinear distance along the line. Other types of parameterization were suggested by Buttenfield (1986) who proposed analysis of the 'structure signatures' of a graphic line in terms of its intrinsic geometry. The preservation of the parametric form of a geographical line during the process of generalization is basically a valid argument but is an approach difficult to implement. The possibility, for instance, of adjusting the tolerance parameters of simplification algorithms by using the parametric description of digital lines has not yet been realized.

Fractal geometry provides another powerful method for describing the nature of complex lines. Mandelbrot (1982) identified two properties of irregular forms – self-similarity and fractal dimensionality. A line is self-similar if a part of the line, when isolated and subsequently enlarged, displays similar characteristics to the whole line. Accordingly, fractal dimensionality, a value which is determined by the complexity of the line, remains unchanged whether the line is displayed at a small or large scale. Hence, the preservation of fractal dimensionality is another criterion which could be used to assess the performance of a generalization algorithm (Muller 1987; Kubik and Frederiksen 1983). This criterion may, however, only be applied when a line is statistically self-similar. Efforts based on heuristic solutions have been applied to the generalization of built-up areas, using classification and template matching methods (Meyer 1987). These heuristic methods are being perfected including automated generalization of Digital Landscape Models (Jager 1987).

All the above techniques are based on mathematical procedures and deal with generalization as if it were exclusively a geometrically rooted problem. Cartographic features are not, however, simply geometrical objects. They have geographical meaning and their significance depends on a variety of factors, such as the map's purpose and user's needs. Hence, we need to look at the substantive content of a database as well.

The rule-based approach

Rule-based methods, as the name indicates, are also a collection of 'IF THEN' statements. These statements relate to symbolic matching rather than string matching, where symbols represent facts of reality. Symbols are always related to data, that is, there is no separation between data and program. Non-procedural or data driven language programs are executed by implementing the rules which represent the relationship between the facts. A rule-based approach implies that our knowledge of the generalization process can be formalized into a chain of reasoning paths, each leading to a particular decision or procedure for generalization to take place. Every reasoning path consists of information involving interdependencies and trade-offs which must be made in order to arrive at a solution. The reasoning process can be expressed naturally by IF..., THEN... or THEN..., IF structures (Jankowski and Nyerges 1989). An example of a simple rule may be:

IF BUILDINGS	(OBJECT, SIZE, DISTANCE, SCALE)
AND OBJECTS	(APARTMENT COMPLEX)
AND SIZE	(SMALL)
AND DISTANCE	(THRESHOLD)
AND SCALE	(1 : 20000)
THEN MERGE	(BUILDINGS)

One could systematize the rules for selection on the basis of user's needs and map functionality. Each single feature in the database could be rated using information requirements identified by user's needs and by examining the relationship of base map elements to thematic features. Necessity factors would be derived for each feature, for each scale, and for each theme (Richardson 1988). Rating matrices could be calculated accordingly, and would function as look-up tables (Fig. 30.5). It is clear that a rule-based approach represents a quantum step beyond the purely algorithmic treatment, including both the tools and the choice of tools to effect generalization.

A catastrophic approach to cartographic generalization

There is a functional relationship between thematic realm, map purpose, scale, and map utility. Cadastral information, *for instance, falls within a* particular range of scales (say between 1 : 5000 and 1 : 10000) *for which map utility is maximized. For another thematic realm, say land use, the optimum map scale may fall between* 1 : 20000 *and* 1 : 100000. The uncertainty about the accuracy of individual observations, and the need to lower random fluctuations and data noise, play an important factor in the determination of scale. Scales which are too large may lead to a representation which is unwarranted by the quality of data sampling or the spatial resolution of the data source. The functional dependence of map utility according to scale, thematic realm, data quality, and map purpose is a topic which requires further study. A simple paradigm may be proposed where utility increases with increasing scales, until a point is reached at which utility starts decreasing (Muehrcke 1969). The curve describing this relationship is probably not a smooth one, as the symbolic representations of cartographic features may change drastically from one scale to another.

That generalization is a phenomenon which involves large variations in the ways nature is abstracted is nothing new. It would be of interest, however, to identify those points where a small variation in scale may cause large variations in the geometrical and substantive content of a map. Those points where a 'catastrophic change' may occur would help us to identify more sharply the range of scales which are suitable for a particular type of map use. The term 'catastrophic' is used here to indicate that the generalization process is discontinuous with respect to scale. It is used as an analogy to catastrophe theory (Thom 1973), although this is concerned with description of the operation of whole systems rather than single elements.

Base map element	Geophysic	Geology	Geomorphology	Climatology	Hydrology	Pedology	Phytogeography	Zoogeography	Ecology	Environment
	Subject realm requirement - 1:2 million									
City	■	●	■	■	●	●	●	●	●	■
Town	?	?	■	■	●	●	?	●	●	■
Village	-	-	●	●	-	?	?	●	●	●
Unincorporated	-	-	?	?	-	?	?	?	-	●
Non-unincorporated	-	-	-	-	-	-	-	-	-	-
Indian reserve	-	-	?	●	●	●	?	?	●	●
Military	-	-	?	●	●	?	-	-	?	?
Rivers	■	■	■	■	■	■	■	■	■	■
Lakes	■	■	■	■	■	■	■	■	■	■
Islands	-	-	-	-	■	●	●	?	?	?
International	■	■	■	■	■	■	■	■	■	■
Provincial	■	■	■	■	■	■	■	■	■	■
Census division	-	-	-	-	-	-	-	-	?	?
Glaciers	?	●	■	●	■	●	■	●	■	■

■ Essential ● Desirable ? Questionable - Unnecessary

Fig. 30.5 Rating of base map elements according to subject realm for national atlas maps at a scale of 1 : 500000 (Richardson 1988: 45).

A distinction must be made between geometrical generalization, which involves essentially simplification, enlargement and displacement, and conceptual generalization, which is effected by selection, classification, typification and symbolization.

It is possible to differentiate between the two classes of generalization by using the classical definition of functional mapping. In one-to-one and *onto transformation*, one-to-one means that distinct elements in map A have distinct images in map B. *In mathematical terms*, F: A->B, that is if $f(a) = f(a')$, $a = a'$. Further, the function F is said to be *onto if every* $b \in$ B *is the image of some* $a \in$ A. A function which is both one-to-one and onto is called *bijective. It is interesting to note that geometrical* generalization (with the subsets simplification, *enlargement and displacement), according to the* definitions above, is both one-to-one and onto. In contrast, conceptual generalization (with the subsets selection, classification and symbolization) is not a bijective transformation. Depending on the cases, it is either one-to-one or onto. Namely:

	one-to-one mapping	onto mapping
Selection	0	1
Classification	0	1
Symbolization	1	0

A conceptual generalization involving all subsets of conceptual transformations is neither one-to-one, nor onto. Such a transformational view of the process of generalization has already been discussed by Morrison (1974). Reformulated in this context, it is possible to argue that a catastrophic change occurs precisely when the transformation process is no longer bijective. A typical topographic map series in the western hemisphere can be used to illustrate this concept.

1. Below and until 1 : 10 000 scales, an isomorphic mapping is applied. The corresponding maps are like topographic plans for which no generalization or only negligible generalization is necessary (Imhof 1937). The map resolution affords the representation of all natural and man-made features of the visible landscape at true scale.
2. Representations at 1 : 20 000 scale provide the first threshold of an abrupt change. Street and road widths are exaggerated; buildings are simplified, combined and displaced; parcels are regrouped and classified into land use categories. Hence, the crossing from isomorphic mapping to generalized mapping signals the first 'catastrophe'.
3. The representations from 1 : 20 000 until 1 : 200 000 scales show objects which are, for the most part, gradually generalized through the process of geometrical, bijective transformations. Rivers and contours are further simplified; settlements are regrouped and exaggerated; rock outcrops are sketched; slopes and hills are shaded; roads are classified and symbolized. The number of classes of objects which are displayed decreases slowly, but the density of objects per square map unit increases. Until now, the representation of the landscape remains fairly realistic, simulating to a large degree what the landscape looks like from an airplane gaining increasing altitude.
4. The next scale in the topographic/geographical map series is 1 : 500 000. At this point, a second 'catastrophe' may be observed. The geometrical form of many spatial objects vanishes and is merely replaced by an abstract or figurative symbol which has little or no resemblance with the original geographical shape. A town, for instance, is simply shown by a circle; an airport is suggested by the sketch of a plane. It is as if geometrical transformations culminate at a point where qualitative (as opposed to quantitative) changes become suddenly necessary. The map has evolved from a realistic to a highly symbolized representation with an increasing predominance of communication road networks and place names. Such a map bears little resemblance with the corresponding landscape photographed from the viewing platform of a satellite flying at an altitude of about 50 miles (assuming a camera focal length of about 15 cm). Generalization is mostly conceptual, controlled by a set of transformational tools whose combination leads to a mapping which is neither one-to-one, nor onto. Note that the major challenges for an automated solution to the problem of generalizing topographic databases come from the middle scale range, between 1 : 20 000 and 1 : 200 000, where both geometrical and conceptual transformations must be combined (Fig. 30.6).

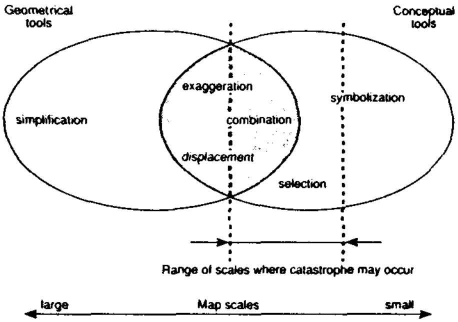


Fig. 30.6 Transformation processes according to map scale. Catastrophe may occur when there is a shift from geometrical to conceptual generalization. Note that some of the procedural tools may be geometrical as well as conceptual. This yields a fuzzy area between the geometrical and conceptual domains.

The threshold point which separates geometrical and conceptual generalization does not occur at the same scale for every cartographic feature and for every thematic realm. Cartographic features of little relevance to the thematic realm, such as a church on a transportation map, may become symbolized sooner than normally expected. In other words, catastrophes occur at different scales for different objects, and are determined by map theme and map function. The relationships between scale, geometrical accuracy, and substantive meaning may be depicted by a manifold of *N* dimensions (*N* object classes) describing the behaviour space of map content (Fig. 30.7). This manifold translated into rules may lead the way towards a truly integrated approach to generalization.

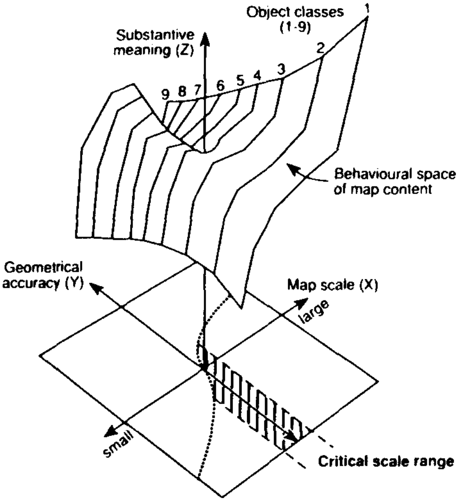


Fig. 30.7 A hypothetical plot describing the relationship between map scale, geometrical accuracy and the substantive meaning of object classes in *x,y,z* space. The manifold has nine dimensions describing the variation in substantive meaning of each of the nine feature classes. A critical scale range has been identified when a fold catastrophe occurs for geometrical accuracy and substantive meaning for feature classes ranging from 5 to 9.

VECTOR- AND RASTER-MODE GENERALIZATION

Vector and raster models express different perceptions of geographical space. Whereas vectors represent geographical objects qualified by their attribute location, raster cells are spatial 'containers' without regard for any objects, attributes, or properties of space within them (Mark and Csillag 1989). Therefore, it is expected that the underlying principles of vector-mode and raster-mode generalization will be different.

Vector-mode generalization

Vector-mode generalization focuses on the simplification, selection and enhancement of linear objects. The lines may be open or closed depending on the topology of the objects they describe (e.g. rivers versus administrative units).

Eighty per cent of all objects (points, lines, and areas) which are found on a typical medium-scale topographic map consist of lines (United Nations 1989). This, in part, explains the considerable interest raised by the issue of line generalization. Numerous algorithms to perform line generalization have been published in the literature (McMaster 1987). Criteria applied to estimate the quality of generalization usually follow the idea of structure preserving transformation. The structure of the line may be characterized by a number of parameters, such as amplitude and density of high- versus low-frequency details and fractal dimension. In the case of self-similar lines, for instance, a non-modifying structure approach to line generalization would preserve fractal dimension (Fig. 30.8). Applying geometrical criteria to evaluate a geometrical approach to generalization leads to a tautology since both methods and judgement are rooted in the same way of reasoning. Surely, the criteria applied to evaluate the generalizations of a shoreline or a political boundary cannot be the same. Instead, non-geometrical criteria derived from geographical meaning and spatial processes must be applied. A purely geometrical approach to line generalization has been increasingly criticized. More sophisticated methods will be required which take into account the phenomenal aspects of a line (whether it is a 'legislated' line or a mathematically derived contour, for example). Furthermore, rules must be provided which can support 'intelligent' decisions for line selection and elimination.

The amalgamation of smaller polygons into larger polygonal units requires a selective elimination of arcs and represents a special case of vector-mode generalization. The operation is based on the statistical generalization of the associated attributes.

The generalization of buildings and built-up areas has received much attention among German cartographers. Efforts have concentrated on the

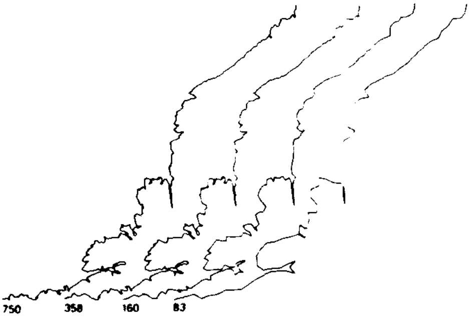
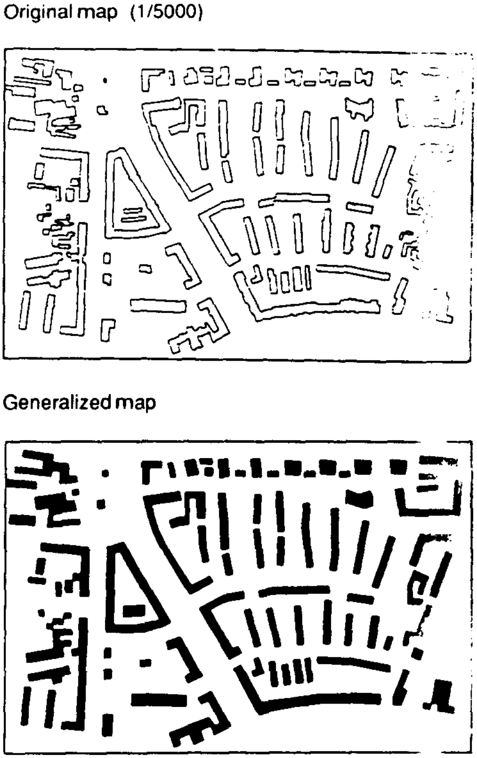


Fig. 30.8 Fractal dimension preserving transformations. The walking generalization algorithm is used to preserve fractal dimension (Muller 1987). Numbers indicate the numbers of points describing the generalized lines.

development of computer-assisted procedures for the generalization of large-scale topographic land survey maps up to about 1 : 5000. The generalization of linear features (e.g. streets) and areal features (e.g. building blocks) can be largely automated (Fig. 30.9). Original attempts were based on geometrically derived algorithmic procedures, such as widening, displacement, simplification and omission (Staufenbiel 1973; Hoffmeister 1978). Recent efforts, however, include structural–conceptual generalization as well as using pattern recognition and expert systems for more complex cartographic generalization models (Grunreich 1985; Powitz and Meyer 1989).

Raster-mode generalization

Whereas vectors lead naturally to an object-oriented approach to generalization, rasters provide the framework for generalization of the attribute component of the data. This is largely a difference of points of view, however, since operationally generalization of objects and their attributes is closely intertwined. One emphasizes data representation while the other is concerned with classification of phenomena. Both affect each other, as has been illustrated in the case of polygon filtering: generalization of the attribute leads to a generalization of the object and vice versa. 'Any treatment of one in isolation from the other will have a high risk of misrepresenting the phenomenon' (Mark and Csillag 1989:68).


Original map (1/5000)

Generalized map

Fig. 30.9 Computer-assisted simplification of building outlines (from Lichtner 1979).

McMaster and Monmonier (1989) recognize four basic classes of raster-mode generalization operators, including structural, numerical, numerical categorization, and categorical. The following review is a close paraphrase of their article. The logical unit of data in a raster model i cell (or resel, Tobler 1984), each of which has an associated set of properties. Most of the raster-based generalization techniques were developed i the fields of digital image processing and terrain analysis using some type of moving kernel or window to filter or smooth regions of an image (see Weibel and Heller 1991 in this volume).

Structural generalization involves a reorganization of raster data where the number of cells is modified while the shape of the cell remain unchanged. Usually (but not necessarily) new larg cells are created through resampling of a grid at a lower level of resolution.

Numerical raster generalization, also termed spatial filtering or convolution, reduces the complexity of an image by smoothing the deviations, or reducing the variance of the matrix. The basic operator is a moving kernel or moving-window mask of weighting factors (Fig. 30.10). Each new value of a cell in the output image is computed by multiplying the original neighbouring values by the corresponding coefficients within the kernel. A new image is created by moving the kernel throughout the original matrix. Numerous types of kernels with different weighting factors have been proposed (Jensen 1986). Some of them are dedicated to image smoothing, whereas others (Laplacian filters) are used for image sharpening. In the latter case, one may consider the process of generalization as one of selection where only the edges and boundaries of an image are represented.

$$\begin{pmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{pmatrix} \begin{pmatrix} 1/10 & 1/10 & 1/10 \\ 1/10 & 1/5 & 1/10 \\ 1/10 & 1/10 & 1/10 \end{pmatrix} \begin{pmatrix} 1/16 & 1/8 & 1/16 \\ 1/8 & 1/4 & 1/8 \\ 1/16 & 1/8 & 1/16 \end{pmatrix}$$

Fig. 30.10 Typical masks for low pass filtering of an image.

The term numerical categorization, also called image classification in the remote sensing literature, is introduced by McMaster and Monmonier to emphasize the process of classification in the context of numerical generalization. The outcome is a reduction of the data – from ratio level to nominal level – and a change in the visual complexity of the image towards a more interpretable product. Three techniques are commonly used for image classification:

1. Minimum distance to means
2. Parallelepiped
3. Maximum likelihood classification

Categorization involves various operations such as the merging of details into more generic categories (e.g. residential, commercial, and industrial lands are collapsed into 'built-up land'), the aggregation of fine grid cells into coarser ones and various attribute-change operators which alter the attributes for selected isolated cells in order to create a map with a simpler structure. For each of those subcategories, there are many ways of performing categorization. For instance, aggregation may regroup cells on the basis of a non-weighted or neighbourhood-weighted kernel (halo bias, Fig. 30.11), or eroding techniques may alter the attributes of the cells differently depending on the decision rules adopted (Fig. 30.12). Furthermore, the combination of eroding and thickening techniques in raster format may be used to simplify areal forms (Lay and Weber 1983). Here, instead of using a structuring element or kernel to determine whether a pixel must be kept or

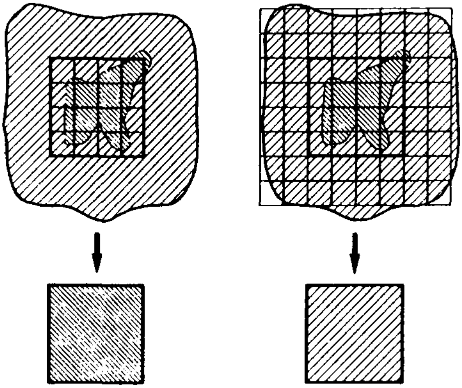


Fig. 30.11 Raster generalization. Finer cells are regrouped into a coarser cell whose attribute is determined by a non-weighted aggregation (left) or a neighbourhood-weighted aggregation (right) (from McMaster and Monmonier 1989).

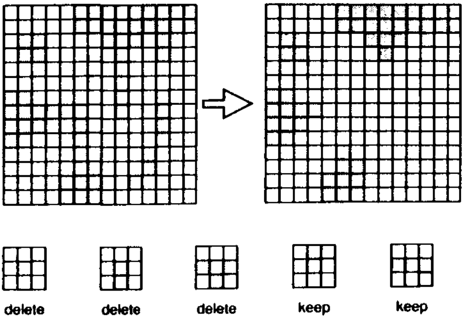


Fig. 30.12 Raster generalization. Erode smoothing (top) based on five decision rules for the moving 3 * 3 kernel (bottom) (from McMaster and Monmonier 1989).

deleted, a blanket of uniform thickness p is added or subtracted at the periphery of a given form. The association between the form A and its characteristic function $K(x)$, may be expressed as:

$$k(x) = 1, x A$$
$$k(x) = 0, x \in A$$

A thickening blanket B is applied to A to yield a new form C:

$$C = U(x + y) \text{ with } x \in A, y \in B$$

An eroding blanket B' of the same thickness p may be subsequently applied to C to yield A':

$$A' = U(x - y) \text{ with } x \in A, y \in B'$$

Generalization occurs when:

$$\underset{y \in B'}{\Sigma k(y)} < \underset{y \in B}{\Sigma k(v)}$$

that is, some pixels of B have been added to A to provide the new generalized form A' (Fig. 30.13 a). It is possible to reverse the operation by using blanket B to erode, and then blanket B' to thicken. Again, generalization occurs if:

$$\underset{y \in B'}{\Sigma k(y)} < \underset{y \in B}{\Sigma k(y)}$$

that is, some pixels of B subtracted from A were not recovered after adding B' (Fig. 30.13b).

Thickening and eroding operations may be repetitively applied in any sequence using different blanket thicknesses in order to arrive at a satisfactory result (Fig. 30.14). These techniques are reminiscent of the epsilon band of Perkal (1965) and could be potentially useful for the generalization of volumetric objects as well.

As mentioned earlier, vectors are boundary oriented, whereas rasters are area oriented. A hybrid approach to generalization would combine those two formats in order to combine their operational powers, possibly at a cost of redundant data storage. The raster strategy is ideally suited for contextual analysis, and would contribute greatly to the vector-based approach to generalization, particularly in the removal of spatial conflicts between vector elements.

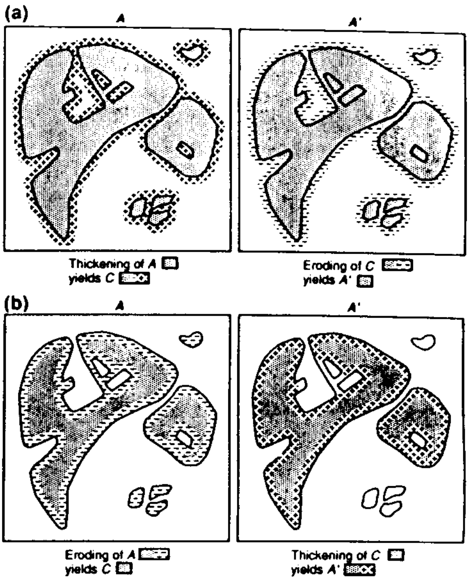
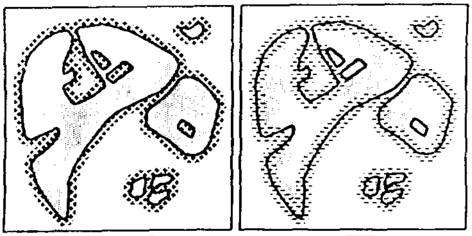


Fig. 30.13 Combinations of thickening/eroding (a) and eroding/thickening (b) techniques for the generalization of areal forms (after Jeworrek 1988). The thickness of the thickening and eroding blankets is held constant. Note that the output A' is very much influenced by the sequence order of the operations.

SCALELESS AND SCALE-DEPENDENT DATABASES

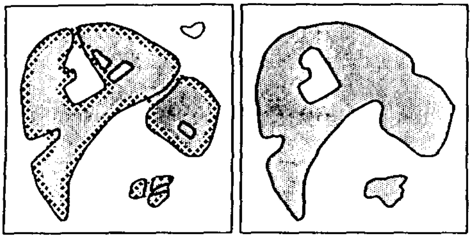
In a scaleless or scale-independent database, the main unit of data collection and revision is set at the level of precision for which the data were captured. Hence, the data are as good as the source. Data collected from ground surveys may be called scale independent, since the notion of scale only appears when those data are being transcribed for analytical or representation purposes, on to a space which is smaller (or larger) than the original surveyed space.

A derived notion for scale-independent databases is the ability to produce cartographic representations at multiple scales from one single source of data, previously referred to as the digital landscape model. Hence, it would be possible to move freely from one level of detail to another appropriate to the scale of display or the precision of data analysis. In order to support multiple



Thickening of A □ by a blanket of thickness p_1 yields C ▨

Eroding of C ▨ by a blanket of thickness p_2, such as $p_2 > p_1$, yields C' □



Thickening of C' □ by a blanket of thickness p_3, such as $p_3 = p_2 - p_1$, yields A' ▨

New image A', generalized representation of the original image A

Fig. 30.14 Thickening and eroding blankets used repetitively with blankets of different thicknesses (examples from Jeworrek 1988).

representations from one single source, an object-oriented data structure combined with generalization operators and decision rules must be implemented. Ideally, it should be possible to operate information extraction and generalization in real time by 'zooming in and out' at any place in any sequence and magnitude. This futuristic notion of scale-independent spatial databases has yet to be realized.

A pseudo-version of the scaleless database concept is a hierarchically structured database in which different scale-dependent layers of representation can be accessed without duplication of data. A pyramidal, multilevel data-adaptive structure to handle points, lines, and areas has been proposed by Jones and Abraham (1986). One obvious application is the hierarchical storage of multiple scale-specific versions of a line. Assuming that the points selected for small scale representations are always a subset of those used in larger scale representations, a structure may be provided which 'reduces the overhead of multiple line storage, while avoiding the access overhead of single, large scale storage' (Jones and Abraham 1986). The structure is a 'multiscale line tree'. When the tree is traversed for whichever level is deemed adequate for the scale requirement, only those points required are accessed. Such techniques merely afford the selection and retrieval of information which is considered relevant for a particular level of resolution, however. Other types of transformation for the purpose of generalization, such as amalgamation and transposition of objects, cannot be performed in this framework.

A third option is the multiple purpose, scale-specific storage scheme. Different maps produced at different scales are stored independently yielding scale-dependent databases.

The advantages and disadvantages of scaleless databases

There are at least three arguments for the development of scaleless databases:

1. *To avoid duplication in storage.* There are basic spatial features, such as coastlines, which may usefully be retained throughout all scale layers. In a scale-dependent environment, they will be stored partly or entirely a multiple number of times.

2. *To allow the production of flexible scale-dependent outputs.* Traditionally scales are fixed according to the standard map series (1 : 10 000, 1 : 25 000, 1 : 50 000, etc.). Any scale 'in between' (say, 1 : 33 333) could theoretically be derived from a scaleless environment. The output could be adjusted to a wide variety of modelling and mapping purposes in order to satisfy specific user needs.

3. *To ensure consistency and integrity between the various scale outputs.* With scale-specific databases, updates must be repeated for every version that has been archived. Update operations may become costly when the information is volatile, such as weather, water supply or transportation networks. There is also the increased chance of inconsistency through errors committed during the propagation of change from one digital representation to another. Object definition may vary according to scale and resolution (an industrial plant may

disappear or become part of an industrial zone) and is difficult to maintain consistently in multiple-scale databases.

There are technical as well as theoretical impediments against scale-independent databases. Among the technical obstacles are the very large overheads incurred every time a smaller scale retrieval occurs. These include both the quantity of data to be accessed and the generalization processing costs to bring the data to a suitable level of resolution. Moreover, solutions to generalization are not fully automated and usually require some further interactive editing before they can be used for output. If the same product is to be generated from a single database several times, the same manual edits will have to be repeated. On the other hand, in the multiple, separate, scale-specific database scheme, retrieval for analysis or mapping applications does not require any processing.

Most mapping agencies have adopted the multiple database strategy, although some are already providing, through adequate database design and topology, the potential for an eventual scaleless approach (Guptill 1989). The major impediment to scaleless cartographic databases is theoretical: database systems will remain limited to the option of multiple scale-dependent representations until appropriate algorithms and rule based programs for the generalization of linear and areal features are made available. This step will require a parallel and holistic approach to generalization, based on spatial relationships and geographical phenomena.

TOWARDS HOLISTIC SOLUTIONS?

Features to be generalized are multiple, including waterbodies, landforms, vegetation, railways, highways, roads and streets, boundary lines, settlements, and so on. Factors influencing generalization are many, including scale, user's requirements, source material, legibility constraints, symbol and colour conventions, technical reproduction capabilities, revision requirements, and so on. Procedural tools to perform generalization are multiple, including simplification, enlargement, displacement, combination, selection, symbolization, and exaggeration. But meaning is also multiple, depending on culture and economical goals. A French town is not simply a town. It is characteri by a geographical background which is differen from an American city. A road is not simply a winding line. Depending on the user, its generalized representation may express a relationship to connectedness or a slope/distance ratio. A phenomenal approach to generalization away fro a purely visual approach has already been advocated elsewhere (Mark 1989). It remains to be seen whether there is an all encompassing model which, like a general-purpose black box, could synoptically relate all features to all factors, tools, and meanings influencing the process of generalization. Would such a universe of formal rules and relationships help us in the practical implementation of programs for batch or semi-batch generalization? Such a model may be intellectually exciting, although it is doubtful whether it is attainable. Practical considerations force us to think in terms of modular processes which are object oriented (one generalization routine to handle names, another to handle lines, etc.) and purpose dependent (settlements will be handled differently whether the map is designed for navigation or tourist purposes). It is doubtful whether generalization programs could even be exported to different countries! (the ratio between lines and built-up areas in the physical/cultural landscape of North America is different from the one in Europe, and would imply a different treatment of the respective objects).

One domain where a holistic view of generalization must be maintained and applied is in the resolution of spatial conflicts. Spatial conflicts arise as a by-product of generalization, as the competition for space increases with a decrease in scale. The resolution of those conflicts requires a simultaneous view of different cartographic features, priorities, and tools. Knowledge-based search and parallel recognition techniques appear to hold the best promises for an automated solution in this field (Mackaness and Fisher 1987). It will be the challenge of future researchers to prove that complex solutions to computer-assisted generalization of spatial databases do not remain an academic issue but can be translated into operational terms.

CONCLUSIONS

This chapter has reviewed the conceptual and technical issues of generalization. It is concluded that generalization is an important issue in the design and operation of GIS. However, the development of robust and widely applicable methods remains as elusive as ever. Potential areas of future research offering most potential are the application of catastrophe theory, the integration of raster and vector approaches and phenomenal generalization.

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