The Development of Analytical Cartography: A Personal Note

Waldo Tobler

In the late 1960s, I initiated a course with the title "Analytical Cartography" at the University of Michigan in Ann Arbor. At the behest of Dr. H. Moellering of Ohio State University (who was at one time a student in the course), a short personal historical perspective of the development of the course was presented at the recent Hawaii meeting of the Association of American Geographers. That review tried to put the subject and the development of the course in the context of the time. This is a written synopsis of the Hawaii presentation.

Some Background

My arrival in Michigan in 1961 followed that of John Nystuen, both of us having come from the University of Washington where our training included the use of computers and the application of quantitative methods to the field of geography. My studies included course work with John Sherman and William Garrison, and with J. Ross MacKay who came to Seattle as a visitor from British Columbia. Charles Davis, then chairman at Michigan, hired me to teach and do research in the field of cartography.

As I began teaching cartography I tried to include some of the subjects I had learned as a graduate student but that were not in the contemporary textbooks. At the time, the choice of such books was restricted to those of E. Raisz and of A. Robinson. Both were excellent books, but not what I was after. Within this context, my course was an attempt to formalize the notion that cartographic methods are used frequently by geographers in their analytical investigations—hence the name "Analytical Cartography," although the course began as "Computer Cartography." What I had in mind could be contrasted with the major treatment in contemporary cartographic teaching and literature as a pure display technique closely related to visual design as practiced in the field of advertising, or in medical illustration. Manual drafting, in ink, or the use of plastic scribing and Zip-A-Tone, were common techniques being taught. But these techniques are not what geographic cartography is all about. We need only to think of the undertakings of Alexander von Humboldt, Alfred Wegener, or Walter Christaller to recognize that significant discoveries are made using maps.

Cartography, in my view, is one of the techniques used by geographers in analytical investigations, along with verbal, statistical, and modeling methods. The purpose of geographical and analytical cartography is the development of geographical theory, not the inventorying of geographical phenomena such as topography or land use, although this may be valuable for practical or applied, non-academic, uses. Such extra-academic uses are not, and never were, of great interest to me. A similar comment would apply to photo interpretation (Tobler 1967; 1968). My emphasis has also always been on the cartography of human geography, and not in the applied physics known as physical geography, and in contrast to what seems to be the topographic map inventorying interest of the majority of cartographers and certainly of government agencies.

The emerging widespread distribution of, and easy access to, digital computers enabled many of the graphical methods used on maps to be recast as mathematical operations. Early on the Swedish meteorological system was producing weather maps by computer. Information theory and Huffman coding also led to a view that geographic information could be measured, transmitted, stored, and analyzed by computers. A lecture describing the "Analytical Cartography" course at the University of Michigan was presented at the University of Vienna in May 1975 while I was on sabbatical leave at the International Institute for Applied Systems Analysis. An English language version of this lecture was subsequently published in the *American Cartographer* (Tobler 1976). These events occurred before the intrusion of deconstructionism into the field.

It is rather obvious that the way in which academics contemplate maps differs from the way in which the general public views maps. Most people do in fact find maps fascinating and useful. Consider, for example, the following two excerpts; the first from Smith (1996), the second from Markham (1983).

Waldo Tobler is Professor Emeritus at the Geography Department of the University of California, Santa Barbara, CA 93106-4060. E-mail: < tobler@geog.ucsb.edu>.

> Blair loved maps. He loved latitude, longitude, altitude. He loved the sense that with a sextant and a decent watch he could shoot the sun and determine his position anywhere on earth, and with a protractor and paper chart his position so that another man using his map could trace his steps to the exact same place, not a second or inch off. He loved topography, the twists and folds of the earth, the shelves that became mountains, the mountains that were islands. He loved the inconstancy of the planet—shores that washed away, volcanoes that erupted from the flat plains, rivers that looped first this way, then that. A map was, admittedly, no more than a moment in the flux, but as a visualization of time it was a work of art.... People could no more resist maps of where they lived than they could portraits of themselves.... Triangulation is the mapmaker's method. If you know the position and height of any two places and you see a third, you can work out its position and height. That's what maps are—invisible triangles (Smith 1996, pp. 39-40).

Markham stresses the fidelity of a map:

> A map in the hands of a pilot is a testimony of a man's faith in other men; it is a symbol of confidence and trust. It is not like a printed page that bears mere words, ambiguous and artful, and whose most believing reader—even whose author, perhaps—must allow in his mind a recess for doubt. A map says to you, "Read me carefully, follow me closely, doubt me not." It says, "I am the earth in the palm of your hand. Without me, you are alone and lost." And indeed you are. Were all maps in this world destroyed and vanished under the direction of some malevolent hand, each man would be blind again, each city be made a stranger to the next, each landmark becomes a meaningless signpost pointing to nothing.
>
> Yet, looking at it, feeling it, running a finger along its lines, it is a cold thing, a map, humorless and dull, born of calipers and a draughtsman's board. That coastline there, that ragged scrawl of scarlet ink, shows neither sand nor sea or rock; it speaks of no mariner, blundering full sail in wakeless seas, to bequeath, on sheepskin or a slab of wood, a priceless scribble to posterity. This brown blot that marks a mountain has, for the casual eye, no other significance, though twenty men, or ten, or only one, may have squandered life to climb it. Here is a valley, there a swamp, and there a desert; and here is a river that some curious and courageous soul, like a pencil in the hand of God, first traced with bleeding feet.
>
> Here is your map. Unfold it, follow it, then throw it away, if you will. It is only paper. It is only paper and ink, but if you think a little, if you pause a moment, you will see that these two things have seldom joined to make a document so modest and yet so full with histories of hope or sagas of conquest. No map I have flown by has ever been lost or thrown away; I have a trunk containing continents. I have the maps I always used *en route* to England and back. I have the log of my flight with Blix (Markham 1983, pp. 245-6).

How Geographers View Maps

Geographers use maps as analytical tools, to help them understand and theorize about the Earth and the phenomena distributed thereon, or to change and modify them. This is the use of maps that I wished to encourage in my students. The materials presented in the paper of 1976 consisted, first, of the status of cartography in the U.S., as I perceived it, followed by an attempt to justify a change in direction. My interpretation of the changing landscape was to assert that "The user of geographic data is, in principle, indifferent as to whether the data are on a geographic map or on a computer tape", and "... when one has enough information on a magnetic tape to be able to draw a geographical map, one also has enough information on that tape to be able to solve all of the problems which could be solved using that map" (Tobler 1976, p. 24).

Several of the more mathematical foundations of cartography were then briefly reviewed. One of these reviews focused on trilateration, now also known to many under the name of multidimensional scaling. Other reviews skimmed over the known facts of geodesy, the shape and size of the Earth, and the mathematics of photogrammetry to the point that students could produce computer stereograms. Analytical photogrammetry was, of course, also rapidly developing at this time. A detailed, week by week course description was presented. I will not repeat this here (it is in the appendix to the 1976 paper), but the description included information on geographic labels (place names), their aliases and conversion between these geographical codings.

We all know that map making consists of a selection and condensation from the immensities of reality to a depiction that presents aspects deemed important. The closely related topic of map generalization was given in examples, including the textual equivalent as produced by the Reader's Digest, the musical equivalent in the overture to an opera, and analogous methods in art and the production of cartoons. It could be shown that some map generalization or filtering methods can be represented by matrix multiplication, and that some of these have inverses, allowing the recovery of the underlying data, as is required of a high fidelity system. Some might even argue that cogent generalization is a large part of the art of developing theoretical constructs or models from reality. This, not the technical details of map generalization, is the important idea.

Cartographic anamorphoses, such as Mercator's projection and area cartograms, provide graphical methods for the solution of problems that are inherently mathematical, as is now well known, and these were dissected and discussed. The similarity of Galton's geographical isochrones to Gaussian polar geodesic coordinates, with radials omitted, was pointed out and explained along with the relevance to transportation systems. Students were also introduced to, and had to program simple problems using computer graphics. Fortunately, the University of Michigan at that time had outstandingly excellent computer facilities. Since movement is the essence of change in geography, we experimented (see Tobler 1970) with animated cartography in the form of computer movies, and students produced class projects such as the growth of Ann Arbor by drawing each street on a CRT screen in temporal order of its having been surfaced, with data from city files. Or they simulated the dynamics of continental drift using world coastal outlines stored, since the early sixties, as vector files on magnetic tape. Computer pattern recognition was advancing under the leadership of Azriel Rosenfeld at the University of Maryland and had obvious applicability to aerial photography and the emerging field of remote sensing. Pattern recognition has great potential for the development of geographical theory, in addition to its application to the routine recognition of objects for inventorying purposes. Two earlier classic papers, *What the Frog's Eye Tells the Frog's Brain* (Lettvin et al. 1959) and McCullogh and Pitts' (1943) *A Logical Calculus of the Ideas Imminent in Nervous Activity* were recognized as important for the field of cartography. The year1966 also saw the publication of Kabrisky's Proposed Model for Visual Information Processing in the Brain; and a few years later Minsky and Papert's (1969) book on *Perceptrons* appeared. These were the sorts of things that students needed to know, and were tested on. MacEachren (1995) seems, based on the publication record, to be the only cartographer who has also kept up with advanced developments in perception and artificial intelligence from that time.

I also discussed some simple GIS functions in the 1976 paper; Roger Tomlinson and Jack Dangermond were active in this nascent field. In the last paragraph of the paper were a few obvious predictions, based on the notion that just because something had not yet been done did not imply that it could not be done. Among the predictions were wristwatch GPS devices and maps on hand-held displays (Tobler 1976, p. 29). All of this was an attempt to give students material that would be of benefit, not for the immediate job market, but for the future. More interestingly, some of the topics are still not well covered in the cartographic literature, the prime example being the sampling theorem—I have included a note on this in the appendix.

Since Then

It is satisfying that several individuals have continued the tradition of analytical cartography. A reasonable question to ask is what have I done since 1976 to enhance the field, and then to look very briefly at what others have done.

In my case there were some papers relating to map projections (Tobler 1977; 1979a; 1986a,b; 1994a; Tobler and Chen 1986; Tobler and Kumler 1991; Yang et al. 2000). More importantly, the partial differential equation governing the entire class of area cartograms, as a generalization of equal area maps, was published (Tobler 1986c). Earlier, computer programs for their computation had been distributed (Tobler 1974). Another important advance, in my opinion, was the development of smooth "pointless" spatial reallocation for the production of density contours (Tobler 1979b). This is an interpolation in which data are not known at points but are given as sums over polygonal regions—for example as the population within census tracts. Maintenance of the sums is the basic fidelity criterion for these maps. This type of spatial reallocation is also useful for the conversion of data from one areal partitioning to another. Recently, a finite element version of the algorithm has become available (Rase 1999). For the quantitative comparison of maps—as for the analysis of old maps in the history of cartography or for rubber sheeting for map matching—my "bi-dimensional regression" (Tobler 1994b) seems useful. A number of studies have also been published on human migration or general movement of people (Tobler 1978a,b; 1979c; 1981; 1982; 1987a,b; 1988; 1995; 1997; Dorigo and Tobler 1983). Later, with

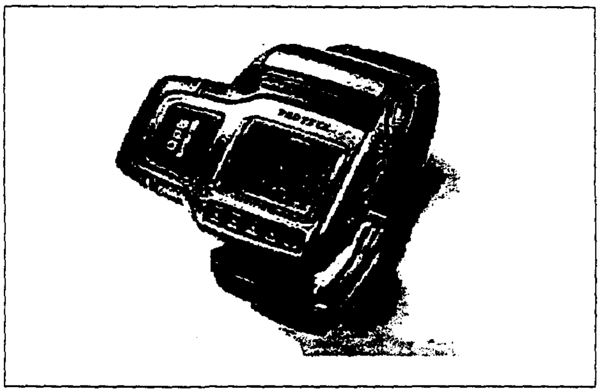

Figure 1. Casio's wrist watch GPS.

the assistance of several students, a compilation has been achieved of world population converted from ephemeral political administrative units to latitude–longitude quadrilaterals and is now available online (Tobler et al. 1997). This compilation allows grid-type analytical studies to be undertaken and comparisons to be made with information assembled using earth satellites.

Other Work

A next question would be to ask what others have done. Obviously not all can be attributed to my paper, but at least three other courses called "Analytical Cartography" were introduced, and one book includes this term in its title (Clarke 1995). Closely related is Dana Tomlin's *Geographic Information Systems and Cartographic Modeling (1990)*, and many more recent books have appeared, including one in a similar spirit by Nick Chrisman (1997). Interactive cartography and animation have become almost routine (Petersen 1995). The global positioning system, and cellular phones, track and keep people and vehicles from getting lost. Computer scientists have developed an intense interest in geographic and cartographic problems and algorithms. Worboys' (1995) book can serve as an example. Cartography of the planets in the near solar system, and even of asteroids, is being explored. The geographical information systems (GIS) industry has become big business, and "time" is now being more explicitly considered in cartography (Langran 1992). Subsurface geology is forcing systems into three dimensions (Raper 1989). *The American Cartographer became Cartography and Geographic Information Science*, and GIS conferences are so numerous and frequent that only large organizations can afford to send attendees to most of them.

"Visualization" has become a "hot" topic (Hearnshaw and Unwin 1994), but cartographers have been doing it all along. What is new is that it is now considered a data reduction technique, somewhat like map generalization, which, incidentally, has turned out to be much more complicated than expected. An academic cartographic software distribution source exists under the auspices of the AAG microcomputer cartography specialty group (http://wolf.its.ilstu.edu/microcam/msg/msg.htm), and the National Science Foundation sponsors the National Center for Geographic Information and Analysis (NCGIA). Computers have become faster and less expensive, ink jet printers have replaced most plotters, and interactive online mapping and routing are now available on the Internet. Digital libraries for geographical information are fully established. Spatial data storage is a growth industry, and the spatial resolution of earth satellites is increasing. The predicted wristwatch GPS is now available from Casio, albeit still with only a barely adequate battery capacity (Figure 1). Garmin now sells, for about $500; a small handheld GPS device containing colored street maps (Figure 2).

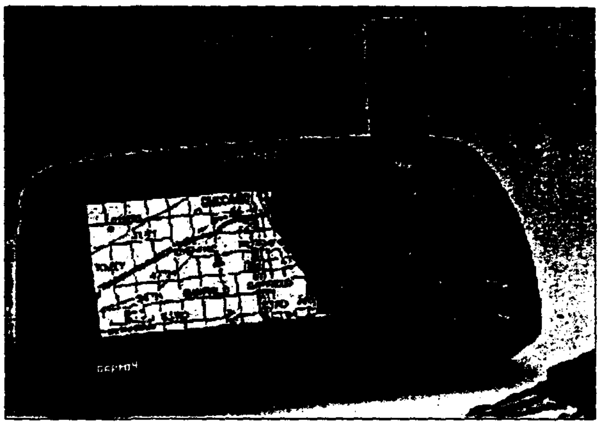

Figure 2. Garvin's handheld GPS with colored map display.

The Future?

What does the future hold? Smaller, faster, less expensive computers are obviously coming, with much increased storage capacities. Five CDs for street maps will be reduced to one DVD. Longer-range forecasts multiply even these capacities by orders of magnitude. I am less sanguine about software improvements but increasingly, algorithms will work over the World Wide Web and data storage may also shift to remote sites. A large percentage of coursework will move to the Web. Real-time problem solving, and perhaps voting, will become available over networks. The magnitude of available geographic data will increase even more dramatically. The cost of this geographic data will also drop significantly, unless the British Ordnance Survey model prevails, in which case the expected economic benefits will be retarded. Will the field of geography benefit, and if so, how? I hope that geographical literacy will improve, and that the deduction of scientific (verifiable) theory will be enhanced along with computational geography. The latter may render analytical cartography obsolete.

Appendix
The Sampling Theorem

The sampling theorem tells one mathematically that in order to represent the full details of a spatial field it is necessary to sample at a rate at least twice as fine as the highest spatial frequency in the field. This means that to capture the delicate detail, sampling must occur at a rate sufficiently fine so that at least two samples capture the object(s) of interest or their separation. Whatever sampling one uses, the highest spatial frequencies that can be contained in the result will not exceed one-half of that rate. This is the Nyquist frequency (Petersen and Middleton 1962; Jaehne 1991:45-52, Baxes 1994:41 et seq).

If sampling occurs at a lower rate than required by the sampling theorem, high frequency details will be missed. In other words, in order to capture a thundercloud the sampling must be at least as dense as half the size of the expected cloud. "At least" is included here because the foregoing statements assume no measurement error. Since there is always measurement error, more detailed sampling is always desirable. Meteorologists generally recommend an oversampling factor of five, even though this increases costs.

The implication here is that if one knows the resolution of a spatial data set, one needs to multiply this by two to estimate the best possible discrimination of detail that can be obtained. Conversely, if one knows the size of the sought detail or object(s), then the minimum required resolution can be specified. The average spatial resolution, in kilometers (or other appropriate units), of geographic data can be estimated using the formula:

$$\text{Average spatial resolution} = \left(\frac{\text{km}^{d}}{\text{Number of observations}}\right)^{1/d}$$

where d is the dimension of the region of interest. In two dimensions, average spatial resolution (in kilometers) is estimated from the square root of the number of square kilometers in the region over the number of observations. The resolution is given as a length. In three dimensions, the cube root must be used to obtain a length.

This equation works for spot observations (e.g., elevations, temperatures taken at specific locations), using the size of the domain as the numerator. For areal observations (census or county aggregations) or pixels, use the area covered by these units. In effect the average spatial resolution measures the average influence domain of each observation, and it can be computed from this notion. The variance of the resolution, computed as the variation of the individual units about their mean, will be a number—small for nearly equally spaced observations (i.e., zero for hexagonal pixels), or large as in the case of U.S. counties. For linear features, the average resolution is obtained by taking the square kilometers of the domain over the total length of the feature (roads, rivers) also estimated in kilometers, to yield a resolution length in kilometers. In two dimensions it is not necessary to extract the root for linear features.

Observe that spatial interpolation can never increase resolution beyond that of the original geographic data, although one can interpolate to arbitrary densities. To see this, think of the spatial frequency content of the data. The resolution of two data sets that have been combined would seem to be that of the coarser data.

The contiguous United States is approximately 5,021,110 square kilometers in size. Thus state data have a resolution of just under 325 km $(5 \text{ x } 10^{6}/50)^{1/2}$, and this should allow the possible detection of objects about 650 km (~400 miles) in size. County data, on the other hand, allow the detection of features 80 km in size, on average. Aggregations of information, say from the U.S. county level to that of the states, of course reduces the resolution, thus obscuring detail. This, therefore, is a type of two-dimensional smoothing filter, but with a complicated and spatially variant response function (Tobler 1969). Modelers beware!

The resolution of a map, in meters, at a particular scale can be estimated by dividing the denominator of the map scale by 2,000 (Tobler 1989, p. 54). Put another way, under ideal conditions one should be able to detect objects on a 1:100,000-scale map which are 100 meters in size (multiplying the calculated 50 meter resolution by two). It's easy to remember—divide by 1,000. Of course, cartographers fudge a little (or a lot), so both more and less can be detected.

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